MULTIPLE CRITERIA DECISION MAKING
Vol. 18 2023

Petr Fiala[*]
Renata Majovská[**]

MULTICRITERIA MODELS IN REVENUE MANAGEMENT

DOI: 10.22367/mcdm.2023.18.02

Received: 30.07.2023 | Revised: 16.09.2023 | Accepted: 15.01.2024.

Abstract

Revenue management (RM) deals with selling the right product to the right customer at the right time at the right price through the right channel to maximise revenue. The innovation of RM lies in the way decisions are made. The performance of revenue management approaches can be evaluated against several criteria. Both discrete and continuous multicriteria models can be used to analyse RM.

The performance pyramid is a comprehensive, fully integrated per-performance system that captures multiple perspectives such as internal, financial, customer and innovation. The assessment is based on a combination of Analytic Hierarchy Process (AHP), Analytic Network Process (ANP) and Data Envelopment Analysis (DEA) approaches.

Customer behavior modeling is gaining increasing attention in revenue management. Customer choice models can be extended with more inputs and more outputs. Evaluation of alternatives can be performed using DEA-based evaluation methods. The search for an efficient frontier in a DEA model can be formulated as a multiobjective linear programming problem. We propose to use an Aspiration Level Oriented Procedure (ALOP) to solve the problem.

Keywords: revenue management, performance measurement, multiple criteria, Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Data Envelopment Analysis (DEA), customer behavior, efficient frontier, Aspiration Level Oriented Procedure (ALOP).

[*] Prague University of Economics and Business, The Faculty of Informatics and Statistics, W. Churchill Sq. 4, 130 67 Praha 3, Czech Republic, e-mail: pfiala@vse.cz, ORCID: 0000-0002-8517-4979.

[**] University of Finance and Administration, The Faculty of Economic Sciences and Management, Prague, Estonská 500, 101 00 Praha 10, Czech Republic, e-mail: renata.majovska@mail.vsfs.cz, ORCID: 0000-0002-9950-8028.

1 Introduction

The general issue is how companies should design their sales mechanisms to maximize expected revenue or profit.

Revenue management (RM) is the process of understanding, predicting, and influencing customer behavior to maximize revenue. The goal of RM is to sell the right product, to the right customer, at the right time, at the right price, and through the right channel to maximize revenue. RM is the art and science of predicting customer demand in real time and optimizing the price and availability of products according to demand. The field of RM encompasses all work related to operational pricing and demand management. It includes traditional problems in this area, such as capacity allocation, overbooking and dynamic pricing, as well as newer areas, such as oligopoly models, negotiated pricing and auctions. Revenue management has seen great success in recent years, particularly in the airline, hotel and car rental industries. Today, more and more industries are exploring the possibility of adopting similar concepts. What is new about RM are not the demand management decisions themselves, but rather how these decisions are made.

The performance pyramid is a comprehensive, fully integrated performance system that captures multiple perspectives, such as internal, financial, customer and innovation. Performance evaluation of RM systems is based on a combination of the Analytic Hierarchy Process (AHP) approach (see Saaty, 1996), Analytic Network Process (ANP) (see Saaty, 2001) and Data Envelopment Analysis (DEA) (see Charnes, Cooper and Rhodes, 1978).

Network revenue management models seek to maximize revenue when customers purchase multiple resource packages. The basic model of the network revenue management problem is formulated as a stochastic dynamic programming problem whose exact solution is computationally difficult. Most approximation methods are based on one of two basic approaches: using a simplified network model or decomposing the network problem into a set of single-source problems. In practice, the deterministic linear programming (DLP) method is popular. The DLP method assumes that demand is deterministic and static. Today's customers actively evaluate alternatives and make decisions. In recent years, there has been interest in incorporating customer choice into these models, further increasing their complexity. Among the effective techniques that have been proposed is the choice-based linear program (CDLP) by Gallego et al. (2004). Mathematical programming models have been developed for revenue management under customer choice (Chen and Homem-de-Mello, 2010). Azadeh, Hosseinalifam and Savard (2015) analyzed the effect of customer behavior models on management systems. Strauss, Klein and Steinhardt (2018) published a review of choice-based revenue management. The development of the science of revenue management continues (Yeoman, 2022).

The contribution of our paper lies in the use of multi-criteria models in revenue management. Both discrete (AHP, ANP, DEA) and continuous (multi--objective LP) multicriteria models can be used for RM analysis. These models can be combined for a detailed analysis of the performance of RM systems.

We focus on finding the efficient frontier of the problem. The efficient frontier provides a systematic framework for comparing different policies and highlights the structure of optimal problem management. The search for the efficient frontier in the model can be formulated as a multi-objective linear programming problem. We propose the Aspiration Level Oriented Procedure (ALOP) method for finding the efficient frontier.

The rest of the paper is organized as follows. Section 2 provides a brief overview of the performance of revenue management systems. Section 3 presents the problems of revenue management in the network. The basic models of customer choice behavior are described in Section 4. The formulation and solution of the efficient frontier search are presented in Section 5. An illustrative example is solved in Section 6. Conclusions are given in Section 7.

2 Performance of revenue management systems

2.1 Revenue management systems

A revenue management system is a specialised information and decision support system. The design of a revenue management system (RMS) includes the core modules, the information flows between modules, and the information provided for decision-making and RM management, such as booking rates and prices. At the core of any RM system are two basic modules, a forecasting module and an optimization module.

The RM process follows four basic steps:

1. Data collection and storage.
2. Forecasting.
3. Optimization.
4. Control.

The first step is to collect and store relevant data on prices, demand and causal factors. The forecasting system attempts to derive future demand based on historical data and current booking activity. The optimization function determines prices and allocations according to demand. Inventory sales management using optimized control is the last step. The objective of RMS is to generate maximum revenue from existing capacity by using different forecasting and optimization techniques. Current RM systems include complex forecasting and optimization models and require accurate information and appropriate actions by RM users for best results. Some factors influencing RM performance are proposed, such as market segmentation, pricing, forecasting, capacity allocation, information technology. Performance systems should capture multiple perspectives such as internal, financial, customer and innovation.

These basic steps of the RM process are repeated, forecasts are refined and the necessary decisions are dynamically optimised to improve the whole process. The structure of the revenue management system is shown in Figure 1.

Several frameworks for measuring performance have been proposed. Several principles emerge from these frameworks. In contrast to the traditional single focus on financial performance, different perspectives need to be taken into account. Many authors have proposed to include non-financial measures in manufacturing performance measurement frameworks alongside traditional cost measures in order to control for the proper execution of manufacturing strategy with respect to all competing priorities (see Kaplan and Norton, 2015; Rouse, Puterill and Ryan, 1997). However, the use of non-financial performance measures makes it difficult to assess and compare the overall effectiveness of individual decision units in terms of the support provided in the implementation of the production strategy, as performance measures expressed in heterogeneous units of measurement need to be integrated to achieve this goal.

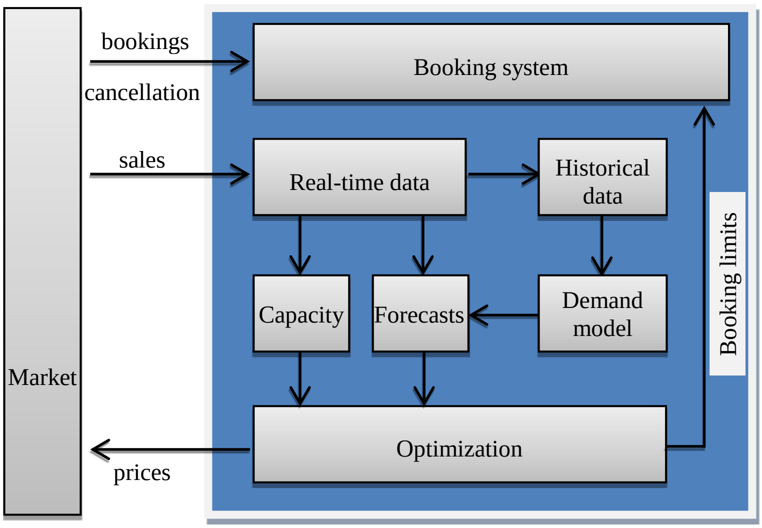

Figure 1: Structure of a revenue management system

Source: Authors.

The Analytic Hierarchy Process (AHP) is a method for prioritization in hierarchical systems (see Saaty, 1996) and the Analytic Network Process (ANP), in network systems. Data Envelopment Analysis (DEA) includes several models and methods for performance evaluation. The performance pyramid is a performance system that captures multiple perspectives (see Rouse, Puterill and Ryan, 1997). We propose to combine these tools to evaluate RM systems.

2.2 Analytic processes

Analytical processes are very popular methods for evaluating and comparing the overall performance of different units. The basic characteristic of these methods is to perform pairwise comparisons of the elements of the system under analysis.

Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a method for prioritization (see Saaty, 1996). The reference-based priority scale is an AHP way to standardize non-uniform scales to combine multiple inputs and multiple outputs and aggregate a hierarchical factor structure. The AHP can be characterized as a subjective weighting method and can be used to weight constraints in DEA.

The AHP derives priorities on a ratio scale by performing pairwise comparisons of elements at a common hierarchy level using a scale of absolute numbers from 1 to 9.

The solution proceeds in three stages:

Stage 1. Creating a hierarchical structure of objectives, criteria and decision options at several different levels with increasing priority up to the highest level. Each level contains parts with similar characteristics to allow comparisons.

Stage 2. At each level of the hierarchy, a pairwise comparison of parts of the system is made. Starting at the top level, a matrix of pairwise comparisons is created and used to estimate the weight vector of each part.

Stage 3: The estimated weights of each part of the system are combined to obtain the aggregated weights and the option with the largest aggregated weight is selected.

The AHP method uses a general model to synthesize performance measures in a hierarchical structure:

$$u_i = \sum_{j=1}^{n} v_j w_{ij} \qquad (1)$$

where u_i is the aggregate weight of the alternative i, v_j are the weights of criterion j, w_{ij} are the weights of alternative i according to criteria j.

Analytic Network Process

The Analytic Network Process (ANP) is a method (see Saaty, 2001) that allows to systematically deal with all kinds of dependencies and feedbacks in a network system. The structure of an ANP model is described by clusters of elements connected by their interdependencies. A cluster groups elements that share a certain set of attributes. At least one element in each of these clusters is associated with an element in another cluster. These connections indicate the flow of influence between elements.

The calculation of the priorities of the system elements takes place in three stages.

Stage 1: Determination of the so-called supermatrix of links between all elements based on pairwise comparison.

Stage 2: Calculation of the so-called weighted supermatrix by multiplying the supermatrix by the cluster weights.

Stage 3: After a certain number of iterations, the powers of the weighted supermatrix are stabilized into the so-called limit matrix. The columns of the matrix will be identical and represent the global priorities of the elements.

2.3 Data Envelopment Analysis

The essential characteristic of the DEA model is the reduction of the multiple input and multiple output using weights to a single ‘virtual’ input and a single ‘virtual’ output. The method seeks a set of weights that maximizes the efficiency of the decision unit. DEA can be characterized as an objective weighting method. The first DEA model was developed by Charnes, Cooper and Rhodes (1978). Various technical aspects of DEA can be found in Charnes, Cooper and Seiford (1994); Cooper, Seiford and Tone (2000); Cooper and Tone (1995).

Suppose there are n decision making units each consuming r inputs and producing s outputs as well as an (r, n) matrix X and an (s, n) matrix Y of observed input and output measures. The essential characteristic of the CCR ratio model is the reduction of multiple input and multiple output to that of a single ‘virtual’ input and a single ‘virtual’ output. For a particular decision-making unit, the ratio of the single output to the single input provides a measure of efficiency that is a function of the weight multipliers (u, v). Instead of using an exogenously specified set of weights (u, v), the method seeks the set of weights which maximize the efficiency of the decision-making unit P_0. The relative efficiency of the decision-making unit P_0 is given as the maximization of the ratio of single output to single input under the condition that the relative efficiency of each decision-making unit is less than or equal to one. The formulation leads to a linear fractional programming problem:

$$\frac{\sum_{i=1}^{s} u_i y_{i0}}{\sum_{j=1}^{r} v_j x_{j0}} \to max$$
$$\frac{\sum_{i=1}^{s} u_i y_{ih}}{\sum_{j=1}^{r} v_j x_{jh}} \leq 1, \text{h} = 1, 2, \ldots, n \tag{2}$$
$$u_i, v_j \geq \varepsilon \text{ , i} = 1, 2, \ldots, s, j = 1, 2, \ldots, r$$

If it is possible to find a set of weights for which the efficiency ratio of the decision-making unit P_0 is equal to one, the decision-making unit P_0 will be regarded as efficient, otherwise it will be regarded as inefficient.

Solving this nonlinear non-convex problem directly is not an efficient approach. The following linear programming problem with new variable weights (μ, ν) that results from the Charnes-Cooper transformation gives optimal values that will also be optimal for the fractional programming problem:

$$\sum_{i=1}^{s} \mu_i \, y_{i0} \to max$$
$$\sum_{j=1}^{r} \nu_j x_{j0} = 1 \tag{3}$$
$$\sum_{i=1}^{s} \mu_i y_{ih} - \sum_{j=1}^{r} \nu_{jh} \, x_{jh} \leq 0, h = 1, 2, \ldots, n$$
$$\mu_{\text{i}}, \nu_{\text{j}} \geq \varepsilon \text{ , } i = 1, 2, \ldots, s, j = 1, 2, \ldots, r$$

If it is possible to find a set of weights for which the value of the objective function is equal to one, the decision-making unit P_0 will be regarded as efficient, otherwise it will be regarded as inefficient.

2.4 Performance pyramid

A wider and more popular performance framework is provided by the balanced scorecard approach of Kaplan and Norton (2015). The performance pyramid (see Rouse, Puterill and Ryan, 1997) builds on the balanced scorecard approach and represents a comprehensive, fully integrated performance system that captures multiple perspectives such as internal business, financial, customer, innovation and learning. The performance pyramid concept is used to evaluate RM systems or their parts. Each side of the pyramid represents a perspective as a hierarchical structure of success factors, managerial measures and process drivers. The hierarchical structure of a pyramid side can be evaluated using the AHP method.

Not only are the measures and process drivers linked to each side of the pyramid, but there are also links to other sides of the pyramid based on the impact of process drivers on multiple key perspectives. The ANP method is used to evaluate this more complex network structure.

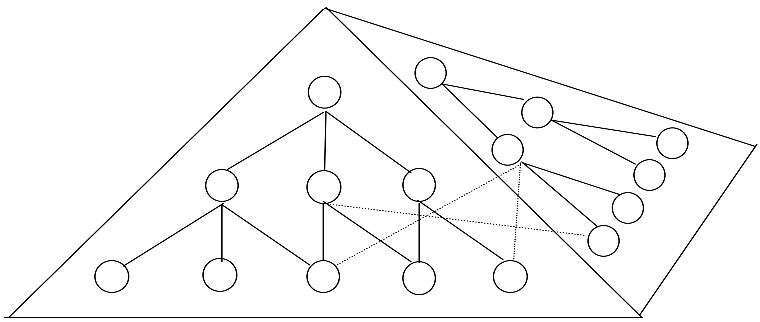

Figure 2: The performance pyramid

Source: Authors.

The efficiency of systems can be measured using the DEA method. The decision maker can restrict the weights in DEA using AHP or ANP. The comparison matrix $C = (c_{jk})$ consists of judgements of w_j / w_k. It is known that the preference region W is structured by column vectors of the comparison matrix C. Any weight vector from W can be obtained as a linear combination of column vectors

$$w = C\lambda \tag{4}$$

where λ is a nonnegative vector of coefficients, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$. If the matrix C is consistent, the consistency index C.I. = 0, the preference region is a line through the origin. If the matrix C is inconsistent, the consistency index C.I. > 0, the preference region is a convex cone; the greater the consistency index, the greater the preference cone.

3 Network revenue management problems

The quantity-based revenue management of multiple resources is referred to as network revenue management. This class of problems arises, for example, in airline, hotel, and railway management. Network revenue management models attempt to maximize a certain reward function when customers buy bundles of multiple resources. The interdependence of resources, commonly referred to as network effects, creates difficulty in solving the problem. The classical technique of approaching this problem has been to use a deterministic LP solution to derive policies for the network capacity problem. A significant limitation of the applicability of these classical models is the assumption of independent demand. In response to this, interest has arisen in recent years to incorporate customer choice into these models, further increasing their complexity (see Talluri and van Ryzin, 2004a; Gallego et al., 2004; Shen and Su, 2007; van Ryzin and Liu, 2008). Because customers will exhibit systematic responses to sales mechanisms, firms are responsible for anticipating these responses when making pricing decisions. The focus is on how customers decide which product to buy in a multi-product revenue management environment. A common approach is to use discrete choice models to capture consumer demand for multiple products. Substitution and complementarity effects for multiple products are also explored. Potential customers do not usually come with a preconceived notion of which product they will buy. Rather, they know only some specific characteristics that a product should have and compare several alternatives that share these characteristics before deciding whether or not to buy.

The basic model of the network revenue management problem can be formulated as follows (see Talluri and van Ryzin, 2004b): The network has m resources which can be used to provide n products. We define the incidence matrix $A = [a_{ij}]$, $i = 1, 2, \ldots, m, j = 1, 2, \ldots, n,$ where:
$a_{ij} = 1$, if resource i is used by product j, and
$a_{ij} = 0$, otherwise.

The j-th column of A, denoted a_j, is the incidence vector for product j. The notation $i \in a_j$ indicates that resource i is used by product j.

The state of the network is described by a vector $x = (x_1, x_2, \ldots, x_m)$ of resource capacities. If product j is sold, the state of the network changes to $x - a_j$. Time is discrete, there are T periods and the index t represents the current time, $t = 1, 2, \ldots, T$. We assume that within each period t at most one request for a product can arrive. Demand in period t is modeled as the realization of a single random vector $r(t) = (r_1(t), r_2(t), \ldots, r_n(t))$. If $r_j(t) = r_j > 0$, this indicates that a request for product j occurred and that its associated revenue is r_j. If $r_j(t) = 0$, this indicates that no request for product j occurred. A realization $r(\text{t}) = 0$ (all components equal to zero) indicates that no request for any product occurred at time t. The assumption that at most one arrival occurs in each period means that at most one component of $r(t)$ can be positive. The sequence $r(t)$, $t = 1, 2, \ldots, T$, is assumed to be independent with known joint distributions in each period t. When revenues associated with product j are fixed, we denote these by r_j and the revenue vector, by $r = (r_1, r_2, \ldots, r_n)$.

Given the current time t, the current remaining capacity x and the current request $r(t)$, the decision is to accept the current request or not. We define the decision vector $u(\text{t}) = (u_1(t), u_2(t), \ldots, u_n(t))$ where:
$u_j(t) = 1$, if a request for product j in period t is accepted, and
$u_j(t) = 0$, otherwise.

The components of the decision vector $u(t)$ are functions of the remaining capacity components of vector x and the components of the revenue vector r, $u(t) = u(t, x, r)$. The decision vector $u(t)$ is restricted to the set:

$$U(x) = \{u \in \{0, 1\}^n, \; Au \le x\} \tag{5}$$

The maximum expected revenue, given remaining capacity x in time period t, is denoted by $V_t(x)$. Then $V_t(x)$ must satisfy the Bellman equation (6):

$$V_t(x) = E\left[\max_{u \in U(x)} \{r(t)^T u(t, x, r) + V_{t+1}(x - Au)\}\right] \tag{6}$$

with the boundary condition:

$$V_{T+1}(x) = 0, \forall x$$

A decision u^* is optimal if and only if it satisfies:
$u_j\,(t, x, r_j) = 1$, if $r_j \ge V_{t+1}(x) - V_{t+1}(x - a_j),\; a_j \le x$,
$u_j\,(t, x, r_j) = 0$, otherwise.

This reflects the intuitive notion that revenue r_j for product j is accepted only when it exceeds the opportunity cost of the reduction in resource capacities required to satisfy the request. The equation (6) cannot be solved exactly for most networks of realistic size. Solutions are based on approximations of various types. There are two important criteria when judging network approximation methods: accuracy and speed. Among the most useful information provided by an approximation method are estimates of bid prices (see Talluri and van Ryzin, 2004b).

Deterministic Linear Programming (DLP) method

The DLP method uses the approximation:

$$\begin{gathered} V_t^{LP}(x) = max\; r^T y \\ Ay \le x \\ 0 \le y \le E[D] \end{gathered} \tag{7}$$

where $D = (D_1, D_2, \ldots, D_n)$ is the vector of demand over the periods $t, t + 1, \ldots, T$, for product j, $j = 1, 2, \ldots, n$, and $r = (r_1, r_2, \ldots, r_n)$ is the vector of revenues associated with n products. The decision vector $y = (y_1, y_2, \ldots, y_n)$ represent partitioned allocation of capacity for each of the n products. The approximation effectively treats demand as if it were deterministic and equal to its mean $E[D]$. The optimal dual variables, π^{LP}, associated with the constraints $Ay \le x$, are used as bid prices. The DLP was among the first models analyzed for network RM. The main advantage of the DLP model is that it is computationally very efficient to solve. Due to its simplicity and speed, it is a popular one in practice. The weakness of the DLP approximation is that it considers only the mean demand and ignores all other distributional information. The performance of the DLP method depends on the type of network, the order in which fare products arrive and the frequency of re-optimization.

4 Customer choice behavior

Customer behavior modeling has been gaining attention in revenue management (see Shen and Su, 2007). Because customers will exhibit systematic responses to the selling mechanisms, firms are responsible for anticipating these responses when making their pricing decisions. The focus is on how customers choose which product to buy in multi-product revenue management settings. A common approach is to use discrete choice models to capture multi-product consumer demand. Substitution and complementary effects across multiple products are also studied. Potential customers usually do not come with a predetermined idea of which product to purchase. Rather, they only know some particular features that the product should possess and compare several alternatives that have these features in common before coming to a purchase or non-purchase decision. This issue of customer choice was first investigated by Talluri and van Ryzin (2004a), who study a revenue management problem under a discrete choice model of customer behavior. There are n fare products, each associated with exogenous revenue r_j, $j = 1, 2, \ldots, n$. At each point in time, the firm chooses to offer a subset of these fare products. Given the subset of offered products, customers choose an option (which may also be a no purchase option) according to some discrete choice model. Gallego et al. (2004), van Ryzin and Liu (2008) extend this analysis to the network setting. Each product consists of a fare class and an itinerary, which may use up resources on multiple legs of the network. The dynamic program of finding the optimal offer sets becomes computationally intractable. The authors adopt a deterministic approximation by reinterpreting the purchase probability as the deterministic sale of a fixed quantity (smaller than one unit) of the product. Under this interpretation, the revenue management problem can be formulated as a linear program, and it is possible to demonstrate that the solution is asymptotically optimal as demand and capacity are scaled up. It is possible to design implementation heuristics to convert the static LP solution into dynamic control policies.

Choice-Based Deterministic LP (CDLP)

The probability that the customer chooses product j given the set of offered fares S (conditioned to arrival of a customer) is denoted by $P_j(S)$. Time is discrete and partitioned into T periods that are small enough so that there is at most one customer arrival with probability λ and no arrival with probability $1 - \lambda$. The network has m resources which can be used to provide n products. The incidence matrix $A = [a_{ij}]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, introduced in network revenue management problems, is used. Demand is treated as known and being equal to its expected value. The problem reduces then to an allocation problem where we need to decide for how many time periods a certain set of products S shall be offered, denoted by $t(S)$. Denote the expected total revenue from offering S by:

$$R(S) = \sum_{j \in S} P_j(S) r_j \tag{8}$$

and the expected total consumption of resource i from offering S by:

$$Q_i(S) = \sum_{j \in S} P_j(S) a_{ij}, \quad \forall i \tag{9}$$

Then the choice-based deterministic linear program (3) is given by:

$$\begin{gathered} V^{CDLP} = \max \sum_{S \subseteq N} \lambda R(S) t(S) \\ \sum_{S \subseteq N} \lambda A P(S) t(S) \leq x \\ \sum_{S \subseteq N} t(S) = T \\ t(S) \geq 0, \quad \forall S \subseteq N \end{gathered} \tag{10}$$

The objective is to maximize total revenue under constraints that consumption is less than capacity and total time sets offered are less than horizon length. Decision variables $t(S)$ are total time periods during which a subset S is offered. There are two basic possible ways to use the CDLP solution. The first one is to directly apply time variables $t^*(S)$ (Gallego et al., 2004). For certain discrete-choice models it is possible to efficiently use column generation to solve the CDLP model to optimality. The solution returns a vector with as many components as there are possible offer sets, and each component represents the number of time periods out of the finite time horizon during which the corresponding offer set should be available. The notion of efficient sets introduced by Talluri and van Ryzin (2004a) for the single leg case is translated into the network context and the authors show that CDLP uses efficient sets only in its optimal solution. The second one is to use dual information in a decomposition heuristic (Liu and van Ryzin, 2007; van Ryzin and Liu, 2008). The dual variables of the capacity constraints can be used to construct bid prices.

5 Searching for the efficient frontier

The models of customer choice can be extended by multiple inputs (input resources, costs, probability of choosing, etc.) and multiple outputs (revenue, profit, output resources, etc.). The evaluation of alternatives can be done by DEA--based evaluation methods. The efficient frontier provides a systematic framework for comparing different policies and highlights the structure of the optimal controls for the problems. Searching for the efficient frontier in the DEA model can be formulated as a multi-objective linear programming problem. We propose an interactive procedure ALOP (Aspiration Levels Oriented Procedure) for multi-objective linear programming problems (see Fiala, 1997). By changing aspiration levels, it is possible to analyze an appropriate part of the efficient frontier.

The set of efficient decision making units is called the reference set. The set spanned by the reference set is called the efficient frontier. Searching for the efficient frontier in the DEA model can be formulated as a multi-objective linear programming problem (see Korhonen, 1997). Suppose there are n decision making units each consuming r inputs and producing s outputs as well as an (r, n) matrix X and an (s, n) matrix Y of observed input and output measures. The problem is defined as maximization of a linear combination of outputs and minimization of a linear combination of inputs.

$$\begin{aligned} Y\lambda &\to \text{"max"} \\ X\lambda &\to \text{"min"} \\ \lambda &\geq 0 \end{aligned} \tag{11}$$

A solution λ_0 is efficient iff there is no other λ such that:

$$Y\lambda \geq Y\lambda_0,\ X\lambda \leq X\lambda_0 \text{ and } (Y\lambda, X\lambda) \neq (Y\lambda_0, X\lambda_0) \tag{12}$$

Different multi-objective linear programming methods can be used for solving the problem.

Aspiration Levels Oriented Procedure

We propose an interactive procedure ALOP (Aspiration Levels Oriented Procedure) for multiobjective linear programming problems (see Fiala, 1997). In the DEA model the decision alternative $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ is a vector of n variable coefficients. The decision maker sets the aspiration levels $y^{(t)}$ and $x^{(t)}$ of outputs and inputs in step t.

We verify three possibilities by solving the problem:

$$\begin{aligned} z = \sum_{i=1}^{s} d_i^+ + \sum_{j=1}^{r} c_j^- &\to \max \\ Y\lambda - d^+ &= y^{(t)} \\ X\lambda + c^- &= x^{(t)} \\ \lambda, d^+, c^- &\geq 0. \end{aligned} \tag{13}$$

If:

- $z > 0$, then the problem is feasible and d^+ and c^- are proposed changes $\Delta y^{(t)}$ and $\Delta x^{(t)}$ of aspiration levels which achieve an efficient solution in the next step,
- $z = 0$, then we obtained an efficient solution,

- otherwise the problem is infeasible, and we search for the nearest solution to the aspiration levels by solving the goal programming problem:

$$z = \sum_{i=1}^{s} (d_i^+ + d_i^-) + \sum_{j=1}^{r} (c_j^+ + c_j^-) \rightarrow \min$$
$$Y\lambda - d^+ + d^- = y^{(t)} \quad (14)$$
$$X\lambda - c^+ + c^- = x^{(t)}$$
$$\lambda , d^+, d^-, c^+, c^- \geq 0$$

The solution of the problem is feasible with changes of the aspiration levels $\Delta y^{(t)} = d^+ - d^-$ and $\Delta x^{(t)} = c^+ - c^-$. For changes of efficient solutions, the duality theory is applied. Dual variables to objective constraints in the problem are denoted q_i, $I = 1, 2, \ldots, s$, and p_j, $j = 1, 2, \ldots, r$.
If:

$$\sum_{i=1}^{s} q_i \Delta y_i^{(t)} + \sum_{j=1}^{r} p_j \Delta x_j^{(t)} = 0 \quad (15)$$

then for some changes $\Delta y^{(t)}$ and $\Delta x^{(k)}$, the value $z = 0$ is not changed and we obtained another efficient solution. The decision maker can set $s + r - 1$ changes of the aspiration levels, and the change of the remaining aspiration level is calculated from the previous equation. The decision maker chooses a forward direction or backtracking. The results of the procedure ALOP are solutions on the efficient frontier.

6 An illustrative example

The individual procedures can be used separately or combined. We demonstrate the use of a certain trivial combination of the ANP, DEA and ALOP procedures. We use the ANP method to determine the most important evaluation indicators. The DEA method will determine the effective units from the population. The ALOP method looks for effective points on the efficient frontier.

We will illustrate the approach to searching for efficient subsets and to improving the proposed price schemes on the following simple example. We use the concept of a performance pyramid with four sides. Each side of the pyramid represents a perspective as a hierarchical structure of success factors, managerial measures and process drivers (Figure 3).

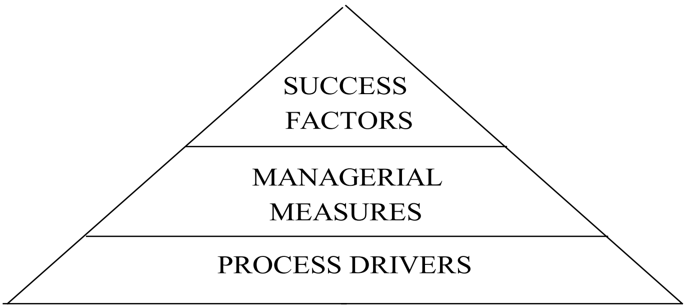

Figure 3: The pyramid side

Source: Authors.

The hierarchical structure of the pyramid side can be evaluated using the AHP method. Since there are links between the elements of different sides of the pyramid, we use the ANP method. The basic relationships within the ANP model are expressed by links between clusters of elements (Figure 4).

First we determine the supermatrix of links between all elements using pairwise comparison. The result of the ANP method is the weights of the process drivers. Due to the number of all elements in our preference pyramid structure, we will not illustrate the numerical solution. We can use these weights in the DEA method. For our example, we will use only the most important indicators for the DEA method evaluation. We assume that the most important indicators are: expected revenues, costs, probabilities of not purchasing.

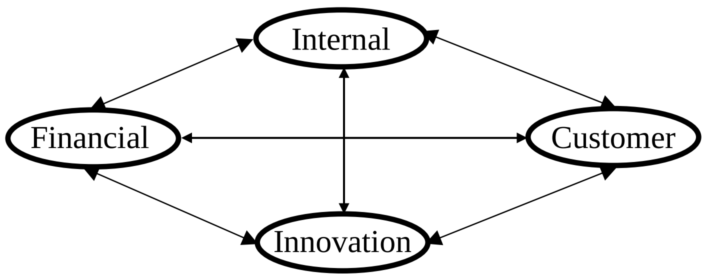

Figure 4: The structure of the ANP model

Source: Authors.

We will use the DEA method. The seller offers nine basic subsets of products $P1$, $P2$, …, $P9$. Expected revenues are taken as outputs, costs are taken as inputs (Input 1). Choice probabilities are considered according to consumer choice behavior. The probabilities of not purchasing are taken as inputs (Input 2). DEA inputs and outputs are summarized in Table 1.

Table 1: DEA inputs and outputs

Product	*P*1	*P*2	*P*3	*P*4	*P*5	*P*6	*P*7	*P*8	*P*9
Output	8	17	30	54	81	90	112	145	182
Input 1	3	8	15	25	35	47	59	72	86
Input 2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

Source: Authors.

By solving the classical DEA model (2), we obtain the score for products. The products *P*1, *P*5, and *P*9 are efficient.

The results are the same as when ALOP is used. Solving the model (13) gives $z = 0$ for efficient units *P*1, *P*5, and *P*9. For other units, the value is $z > 0$ and ALOP gives the proposed changes of aspiration levels for inputs and output for which we obtain efficient units. The results of the ALOP approach are summarized in Table 2.

Table 2: ALOP results

Product	*P*1	*P*2	*P*3	*P*4	*P*5	*P*6	*P*7	*P*8	*P*9
Score	1.00	0.85	0.83	0.92	1.00	0.86	0,87	0.94	1.00
$\boldsymbol{d^+}$	0.00	3.10	6.30	4.65	0.00	15.06	17.14	9.78	0.00
$\boldsymbol{\lambda_1}$	1.00	1.50	1.50	0.75	0.00	0:00	0.00	0.00	0.00
$\boldsymbol{\lambda_5}$	0.00	0.10	0.30	0.65	1.00	0.81	0.62	0.35	0.00
$\boldsymbol{\lambda_9}$	0.00	0.00	0.00	0.00	0.00	0.22	0.43	0.70	1.00

Source: Authors.

The efficient products are offered to customers. The ALOP procedure is used for detailed analysis of the efficient frontier and for searching for better price schemes. For example, we start with efficient unit *P*5 and search for the efficient frontier. The decision maker sets the aspiration levels of output and inputs $y^{(1)} =$ $= 90$, $x_1^{(1)} = 40$, $x_2^{(1)} = 0.5$. Model (13) is infeasible for these aspiration levels, therefore ALOP searches for the nearest solution to the given aspiration levels by solving the goal programming model (14). It proposes $\Delta x_2^{(1)} = 0.0218$, and the new aspiration levels $y^{(2)} = 90$, $x_1^{(2)} = 40$, $x_2^{(2)} = 0.5218$ correspond to the efficient point on the efficient frontier.

7 Conclusions

Revenue management is the process of understanding, anticipating and influencing customer behavior to maximize revenue. Revenue management problems can be modeled by multicriteria models. The paper proposes an approach to performance evaluation, based on a combination of AHP, ANP, DEA approaches and the concept of performance pyramid. A more insightful view may be obtained by separating out measures of efficiency, effectiveness and economy (the concept of the three "E's"). Efficiency can be expressed in terms of the relationship between outputs and inputs, effectiveness in terms of the relationship between outputs and outcomes, and economy in terms of the relationship between outcomes and inputs.

Network revenue management models attempt to maximize revenue when customers buy bundles of multiple resources. The basic model of the network revenue management problem is formulated as a stochastic dynamic programming problem whose exact solution is computationally intractable. The popular Deterministic Linear Programming (DLP) method assumes that demand is deterministic and static. The common modeling approaches assume that customers are passive and do not engage in any decision-making processes. This simplification is often unrealistic in practice. In an effort to incorporate customer choice into these models, we analyze strategic customer behavior. The customer's choice depends critically on the set of available products. A modeling approach for strategic customer behavior based on deterministic linear programming (CDLP) was investigated. Our paper introduces the multicriteria model to search for the efficient frontier and proposes the ALOP method to solve it.

A combination of methods for searching for the efficient frontier and methods for specific requirements (weight restrictions, aspiration level changes) gives a powerful instrument to approach revenue management problems.

Acknowledgment

This work was supported by grant No. IGA F4/42/2021, Faculty of Informatics and Statistics, Prague University of Economics and Business.

References

Azadeh S.S., Hosseinalifam M., Savard G. (2015), *The Impact of Customer Behavior Models on Revenue Management Systems*, Computational Management Science, 12, 99-109.

Charnes A., Cooper W.W., Rhodes E. (1978), *Measuring Efficiency of Decision-Making Units*, European Journal of Operational Research, 2(6), 429-444.

Charnes A., Cooper W.W., Seiford L.M. (1994), *Data Envelopment Analysis: Theory, Methodology, and Applications*, Kluwer Publ., Boston.

Chen L., Homem-de-Mello T. (2010), *Mathematical Programming Models for Revenue Management under Customer Choice*, European Journal of Operational Research, 203, 294-305.

Cooper W.W., Seiford L.M., Tone K. (2000), *Data Envelopment Analysis*, Kluwer Publ., Boston.

Cooper W.W., Tone K. (1995), *A Survey of Some Recent Developments in Data Envelopment Analysis*, Proceedings of the EURO XIV Conference, Jerusalem, 149-168.

Fiala P. (1997), *Models of Cooperative Decision Making* [in:] T. Gal, G. Fandel (eds.), *Multiple Criteria Decision Making*, Springer, Berlin, 128-136.

Gallego G., Iyengar G., Phillips R., Dubey A. (2004), *Managing Flexible Products on a Network*, Tech. Rep. TR-2004-01, Department of Industrial Engineering and Operations Research, Columbia University.

Kaplan R.S., Norton D.P. (2015), *Balanced Scorecard Success: The Kaplan-Norton Collection (4 Books)*, Harvard Business Review Press.

Korhonen P. (1997), *Searching the Efficient Frontier in Data Envelopment Analysis*, IIASA Report IR-97-79, IIASA, Laxenburg.

Liu Q., van Ryzin G.J. (2007), *Strategic Capacity Rationing to Induce Early Purchases*, Working Paper, Columbia University.

Rouse P., Puterill M., Ryan D. (1997), *Towards a General Managerial Framework for Performance Measurement: A Comprehensive Highway Maintenance Application*, Journal of Productivity Analysis, 8, 127-149.

van Ryzin G.J., Liu Q. (2008), *On the Choice-based Linear Programming Model for Network Revenue Management*, Manufacturing & Service Operations Management, 10, 288-311.

Saaty T.L. (1996), *The Analytic Hierarchy Process*, RWS Publications, Pittsburgh.

Saaty T.L. (2001), *Decision Making with Dependence and Feedback: The Analytic Network Process*, RWS Publications, Pittsburgh.

Shen Z.-J., Su X. (2007), *Customer Behavior Modeling in Revenue Management and Auctions*, Production and Operations Management, 16, 713-728.

Strauss A.K., Klein R., Steinhardt C. (2018), *A Review of Choice-based Revenue Management: Theory and Methods*, European Journal of Operational Research, 271, 375-387.

Talluri K., van Ryzin G.J. (2004a), *Revenue Management under a General Discrete Choice Model of Consumer Behaviour*, Management Science, 50, 15-33.

Talluri K.T., van Ryzin G.J. (2004b), *The Theory and Practice of Revenue Management*, Kluwer Publ., Boston.

Yeoman I. (2022), *The Continuing Evolution of Revenue Management Science*, Journal of Revenue and Pricing Management, 21, 1-2.