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Applying Benford's law to detect earnings management

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Abstract

Aim/purpose – This paper analyzes the role of Benford's law in the detection of earnings management in Poland. Previous research that uses Benford's law does not split the sample into a fraud and a control group; however, this method is used in logistic regression and data mining analysis.

Design/methodology/approach – The sample comprises 126 observations of Polish non-financial companies listed on the Warsaw Stock Exchange for the years 2010-2021. The author uses first, second, and first-two digits analysis as a proxy for earnings management detection.

Findings – The results indicate that fraudulent companies have different deviations in the digits than control firms. Accordingly, the statistical test results indicate that control companies have weaker conformity with the Benford distribution than fraudulent companies.

Research implications/limitations – The study sample is limited to 126 observations, which is due to the small number of listed firms that received a monetary fine from the Polish Financial Supervision Authority (UKNF Board) for violation of IAS/IFRS principles related to their financial statements during the study period.

Originality/value/contribution – The author offers a significant contribution to the accounting literature by proposing the separation of fraudulent and control observations in Benford analysis due to differences in the deviations of digits. Also, analyzing the full sample may lead to the identification of inappropriate areas for further auditor analysis.

Keywords: earnings management, digital analysis, Polish companies.
JEL Classification: C46, M40, M42.

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1. Introduction

There is increasing concern in financial markets regarding the quality of the financial statements issued by public companies. As companies become larger, their financial statements become more complex, and as a consequence, fraud detection becomes more difficult and more expensive. Proactive access restrictions and post facto forensic accounting procedures are widely employed to protect enterprises from losses. However, audits usually focus only on the areas that are more exposed to risk, and this may contribute to fraud.

The manipulation of financial data leads to the falsification of financial statements. This manipulation adversely affects the economic decisions of external users of financial statements. That is why the detection of earnings manipulation is important. Detection of fraud in financial statements requires more advanced procedures than just standard audit procedures (Asllani & Naco, 2014).

Auditors often use Benford analysis (Durtschi et al., 2004; Morales et al., 2022; Nigrini, 2022) to identify irregularities in large datasets. Benford's law is perhaps the simplest and most commonly known but still effective test. The Benford distribution is a logarithmic distribution that decays as the number of digits increases. Benford analysis is used to investigate financial data irregularities and test the data for conformance with the distribution. The comparison between the observed and expected probability can indicate the occurrence of fraud and errors when significant deviations are detected. Authors who have used Benford's law (Mataković, 2019; Nigrini, 2005; Zdraveski & Janeska, 2021) have shown that the distribution of first digits in financial reports complies with the law in most cases. However, if there is a falsification of financial statements, then the distribution of the first digits changes. As a result, the likelihood of falsification of financial statements can be verified statistically, and, at least theoretically, the reliability of a specific financial report can be quantified.

This article explores the potential of Benford's law in the detection of earnings management in Poland. Previous research that used Benford's law did not split the sample into a fraud and a control group; however, this method is used in logistic regression and data mining analysis. Digital analysis can assist auditors in identifying cases in which fraud might occur so that they can analyze data more accurately. The analysis is based on a sample of 63 observations of public companies listed on the Warsaw Stock Exchange (WSE) that were involved, according to the Polish Financial Supervision Authority (UKNF Board), in alleged instances of earnings management over the period of 2010-2021. Each fraudulent company was matched with a control firm based on firm size, financial year, and industry. The hypothesis of the study is as follows:

H1: There are differences between the distributions of the first, second, and first two digits of figures reported in the financial statements of fraudulent and control companies.

This study contributes to the literature on the detection of financial statement fraud and Benford's law in several ways. First, the division of the sample into a fraud and a control group is used in traditional methods to detect earnings management, such as logistic regression or data mining methods, but is omitted in studies using only Benford's law. Second, the results show that fraudulent and control firms have other identified differences in digit distribution compared to the full sample. Other differences from Benford's law for the fraudulent and control samples indicate that the auditor should check different values that may be manipulated by the company, rather than the values obtained for the full sample. Third, the results of the statistical tests for the fraudulent companies show closer conformity with the Benford distribution than the control companies. This may indicate that the fraudulent companies are more careful about complying with Benford's law, so as not to signal that additional tests are needed. Finally, the results for individual elements of the financial statements indicate that closer conformity in all elements applies to fraudulent companies.

The rest of this paper is organized as follows. Section 2 contains the literature review. Section 3 describes the research methodology used in the analysis. Section 4 presents the results of the analysis. Section 5 discusses the relevance of the research findings and Section 6 provides the conclusion of the study.

2. Literature review

Benford (1938) was convinced that more numbers have small leading digits than have large leading digits. Empirical studies led Benford to propose that, in many real-world applications, the first digits D_1, the second digits D_2, and the first two digits D_3 follow the probability distribution below:

$$P(D_1 = d_i) = \log\left(1 + \frac{1}{d_i}\right), \text{ for } (d_i = 1, 2, \ldots, 9) \quad (1)$$

$$P(D_2 = d_i) = \sum_{j=1}^{9} \log\left(1 + \frac{1}{10*j + d_i}\right), \text{ for } (d_i = 0, 1, \ldots, 9) \quad (2)$$

$$P(D_3 = d_i) = \log\left(1 + \frac{1}{d_i}\right), \text{ for } (d_i = 10, 11, 12, \ldots, 99) \quad (3)$$

Diaconis and Freedman (1979) showed that Benford manipulated rounding errors to obtain a better fit for the logarithmic law. However, Hill (1995) found that digits distribution is logarithmic. For this reason, Benford's law is very popular and has given rise to much different research (Berger & Hill, 2015).

Prior studies have investigated whether financial accounting data follow Benford's law. Carslaw (1988) examined the frequency of occurrence of the second digit of the profit in a sample of New Zealand firms. The author found that there are significantly more 0s and fewer 9s than expected. Carslaw (1988) explained that this result is because managers tend to round up the firm's profit to improve its situation.

Nigrini (1994) demonstrated that Benford's law could be used in fraud detection. The author showed that the first two digits of payroll data can be considered as numbers that reveal fraud and deceit when they deviate significantly from Benford's law. Nigrini (1996) also used the first and second-digit tests to detect tax evasion. Low-income taxpayers were more likely to fabricate numbers on their tax returns. In similar research, Nigrini and Mittermaier (1997) and Nigrini (2000) showed that digital frequencies are well suited to identifying manipulation in financial statements. Nigrini (2005) also used Benford's law to identify wide-scale earnings management in the period around the Enron crisis. Saville (2006) showed that South African companies' income statement data fail Benford's law for all fraud observations.

Durtschi et al. (2004) explained that digital analysis is performed on transaction-level data rather than aggregate or combined data, so it can assist auditors in identifying potential fraud in financial data. Hales et al. (2008) claimed that Benford's law is a cost-effective and easily applicable method compared with methods such as statistical sampling. Furthermore, Benford's law is considered, along with other techniques, to be orientated toward the detection of earnings management. In some studies, authors have used various values of net financial results in digital analyses to detect financial fraud (Özevin et al., 2020; Skousen et al., 2004; Žgela & Dobša, 2011). Other researchers have used other variables, such as net sales (Bader & Saleh, 2017; Özarı & Ocak, 2013), receivables (Máté et al. 2017), inventory (Tilden & Janes, 2012), allowance for doubt (Tilden & Janes, 2012), earnings per share (Kumar et al., 2018; Roxas, 2011) and total assets (Istrate, 2019; Jordan et al., 2009), to detect deviations of digits distributions from selected positions according to Benford's law. In contrast, some studies (Amiram et al., 2015; Baryła, 2017; Saville, 2006) used all positions of financial statements. Digital analysis does not prove the existence of fraud but indicates the need for further investigation of the company's financial information.

3. Research methodology

A company was identified as an instance of alleged earnings management when it had received a monetary fine from the UKNF Board in the context of compliance with International Accounting Standards (IAS) or International Financial Reporting Standards (IFRS) principles. The sample included 63 observations of public companies listed on the WSE involved in alleged instances of earnings management during the period of 2010-2021. In over 25% of cases, the UKNF has fined the company for failing to provide liquidity risk disclosures and for the lack of analysis of the maturity dates of financial assets held for liquidity risk management related to IFRS 7. In approximately 20% of cases, the UKNF has fined a company for not including all subsidiaries of the parent in consolidated financial statements under IAS 27 or for the lack of assessment of an entity's ability to continue as a going concern under IAS 1. In more than 10% of cases, the fines were related to failure to disclose merger amounts under IFRS 3 or failure to provide objective evidence of assets impairment under IAS 39. However, based on the independent auditor's reports of the financial statements, in more than 30% of cases, there were raised objections to the assumption of going concern.

The 63 fraudulent observations were matched with 63 control observations. Matched pairs of samples were used, whereby each company was matched with a corresponding control firm based on:

– Firm size, where a non-fraudulent firm was considered similar if total assets were within +/–30% of the total assets of the fraudulent firm in the fraud year.
– Financial year, where annual reports for non-fraudulent firms were available for the same year as for the fraudulent firm.
– Industry, where firms were reviewed to identify a non-fraudulent firm within the same three-digit Standard Industrial Classification (SIC) as the fraudulent firm. If no match was found, two-digit codes were used.

The financial data expressed in monetary units were collected from the annual financial statements of public companies. The analysis included all data from the four basic elements of financial statements: Balance Sheet, Statement of Income, Statement of Cash Flow, and Statement of Changes in Equity. The initial sample included 15 120 records for fraudulent companies and 14 618 records for non-fraudulent companies. Furthermore, additional observations for which the records showed smaller amounts than 10.00 were excluded because these numbers are usually immaterial from an auditing perspective. This resulted in a total sample of 14 863 records for fraudulent companies and 14 366 records for non-fraudulent companies. This means that for each fraud observation, there were 236 average financial positions per year and for each control observation, there were 228 average financial positions per year.

Comparing actual data with the expected Benford's law numbers involves testing the first, second, and first two digits. The first-digit test is an initial test of reasonableness, while the second and first-two digits tests are more effective tests of deviation from Benford's law to identify potential manipulation (Alali & Romero, 2013). The critical values of ranges for the first, second, and first two digits were developed by Drake and Nigrini (2000) and next updated by Nigrini (2020). The existing literature suggests many methods of testing the conformity of digits. However, the three tests often used are the mean absolute deviation (MAD) test (Johnson, 2009; Máté et al., 2017; Nigrini, 2020), the Kolmogorov–Smirnov (KS) test (Amiram et al., 2015; Miller & Nigrini, 2008; Nigrini, 2015), and Z-statistics (Guan et al., 2008; Mataković, 2019; Tilden & Janes, 2012).

The MAD test is independent of the sample size. The formula is calculated as follows:

$$MAD = \frac{1}{n} * \sum_{i=1}^{9} |AP - EP| \tag{4}$$

where n is the number of digits, AP is the actual proportion and EP is the expected proportion.

Nigrini (2020) used the MAD test to classify the dataset conformity with Benford's law as one of four types of results: close conformity, acceptable conformity, marginally acceptable conformity, and nonconformity. The critical value and ranges are presented in Table 1.

Table 1. The critical values and conclusions for various MAD values

Conformity range	First digits	Second digits	First two digits
Close	0.000-0.006	0.000-0.008	0.0000-0.0012
Acceptable	0.006-0.012	0.008-0.010	0.0012-0.0018
Marginally acceptable	0.012-0.015	0.010-0.012	0.0018-0.0022
Nonconformity	Above 0.015	Above 0.012	Above 0.0022

Source: Nigrini (2020).

However, Özevin et al. (2020) proposed the Benford Digit Score (BDS). The BDS means taking the average of the MAD values calculated for the three-digit values: the first, second, and first two digits.

$$BDS = \text{average}\begin{pmatrix} MAD\ for\ first\ digit, MAD\ for\ second\ digit, \\ MAD\ for\ first\ two\ digits \end{pmatrix} \tag{5}$$

Özevin et al. (2020) suggested BDS conformity criteria for "close" and "acceptable" of 0.0000-0.0095 and 0.0095-0.0157, respectively. A BDS value above 0.0157 is considered to indicate nonconformity.

Kossovsky (2014) proposed using a Sum of Squared Deviations (SSD), which is an empirically-based whole-test measure that simply takes the sum of the squares of the deviation of each digit's frequency from Benford's law frequency. Specifically, this is given by:

$$SSD = \sum_{i=1}^{n}(AP_i - EP_i)^2 * 10^4 \tag{6}$$

The critical value and ranges for the SSD test proposed by Kossovsky (2014) are presented in Table 2.

Table 2. The critical values and conclusions for various SSD values

Conformity range	First digits	Second digits	First two digits
Close	0-2	0-2	0-2
Acceptable	2-25	2-10	2-10
Marginally acceptable	25-100	10-50	10-50
Nonconformity	Above 100	Above 50	Above 50

Source: Kossovsky (2014).

Barney and Schulzke (2016) proposed replacing MAD with excess MAD for the first two digits test, which explicitly adjusts for sample size in estimating deviation from Benford's law. The excess MAD is the difference between MAD and expected MAD (E(MAD)) values and represents the portion of the observed MAD above that is attributable to chance alone. The formula for E(MAD) is calculated as follows:

$$E(MAD) = \sum_{k=10}^{99}\sum_{j=0}^{N}\binom{N}{j}(p_k)^j(1-p_k)^{N-j}\frac{\left|\left(\frac{j}{N}\right)-p_k\right|}{90} \tag{7}$$

where $p_k = \log_{10}(1+\frac{1}{k})$ and for k =10, 11, 12,…,99. If the sample size is larger than 500, then E(MAD) is approximated by:

$$E(MAD) \approx \frac{1}{\sqrt{158.8N}} \tag{8}$$

If excess MAD is less than zero, then there is evidence of conformity. However, if excess MAD is greater than zero, then it is a measure of nonconformity.

The KS test is a nonparametric statistical test that uses a maximum deviation of digits from Benford's law. The deviations in the KS test are defined as absolute cumulative differences between the observed and expected probabilities of the digits. In the KS test, the null hypothesis assumes that the data follow the specified distribution, and the alternative hypothesis assumes that the data do not follow the required distribution. The formula for the KS statistic is calculated as follows:

$$KS = \max|F(AP_i - EP_i)| \tag{9}$$

where $F(.)$ stands for cumulative relative frequencies.

For the KS test, a value of 1.36 at the 5% significance level is often used to calculate the test critical value, but the function must have a normal distribution (Davis & Stephens, 1989). Benford's distribution is discrete; therefore, a value will be equal to 1.148 (Morrow, 2014). The critical value of the KS statistic to test whether a dataset conforms to Benford's law is calculated as follows:

$$\frac{1.148}{\sqrt{N}} \tag{10}$$

However, the KS test has one major disadvantage that the result can be very sensitive to sample size. In this case, the KS test becomes sensitive because the test value is calculated based on the total number of digits. Therefore, a modified KS test, the Kuiper test (Kuiper, 1960), can be used because it ignores the sample size. It is based on the following formula:

$$V = (D_+ + D_-) * (\sqrt{N} + 0.155 + 0.24 * \sqrt{N}) \tag{11}$$

where $D_+ = \max[\mathrm{F(AP_i)} - \mathrm{F(EP_i)}]$ and $D_- = \max[\mathrm{F(EP_i)} - \mathrm{F(AP_i)}]$.

The discrepancy statistics D_+ and D_- characterize the absolute sizes of the differences between the two distributions being compared: the absolute and the observed distribution.

For the Kuiper test, a critical value of 1.747 at the 5% significance level is often used to indicate whether a dataset follows Benford's law, but the function must have a normal distribution (Davis & Stephens, 1989). Benford's distribution is discrete, so the critical value will be equal to 1.321 (Morrow, 2014).

Some authors (Kossovsky, 2021; Tam Cho & Gaines, 2007; Tsagbey et al., 2017) oppose using statistical tests to assess compliance of data conformity with Benford's distribution due to their excess power in the large datasets. In these cases, Miller and Nigrini (2008) proposed further advanced tests for large datasets to clarify (non)conformity. In the analysis, the researchers also included the Z-statistic test, which is used to determine whether the actual proportion for a specific digit combination differs significantly from the expectation of Benford's distribution. The Z-statistic formula is calculated as follows:

$$Z = \frac{|AP - EP| - \frac{1}{2N}}{\sqrt{\frac{EP*(1-EP)}{N}}} \tag{12}$$

where *N* is sample size. The (1/2N) term is only used when the sample is smaller than the first term in the numerator. The calculated value of the Z-statistic is compared at a certain significance level; that is, at the 5% level, Z is equal to 1.96.

In addition to the indicated tests, the Pearson chi-square statistic (Alali & Romero, 2013; Nigrini, 2020) and the Euclidean distance (Máté et al., 2017; Tam Cho & Gaines, 2007) are also often used for Benford's law compliance analysis. The Pearson chi-squared statistic is used to compare the actual sample results with the expected set of results. The Euclidean distance test compares the distance between the observed distribution and Benford's distribution. The formula for the Pearson chi-square statistic and the Euclidean distance are calculated as follows:

$$Chi - square = \sum_{i=1}^{n} \frac{(AP-EP)^2}{EP} \tag{13}$$

$$Euclidean\ distance = \sqrt{\sum_{i=1}^{n} (AP - EP)^2} \tag{14}$$

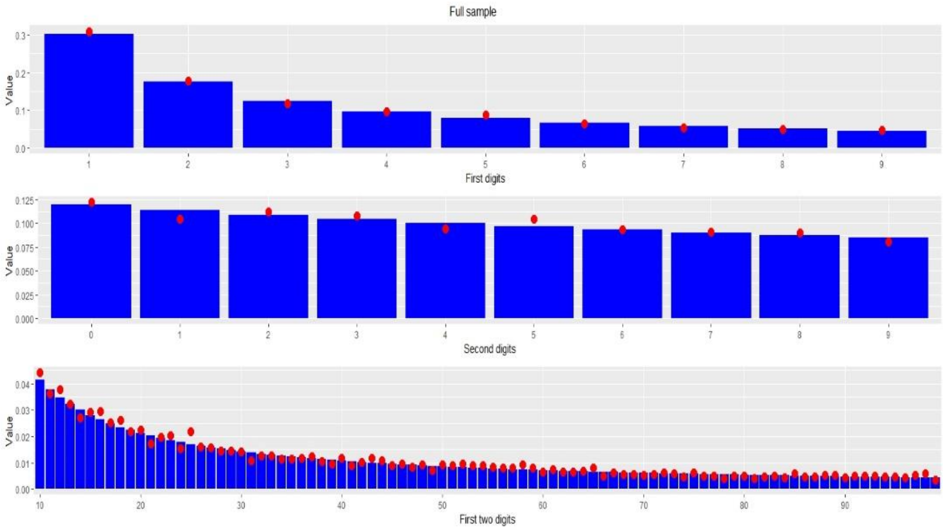
The results of the selected statistical tests should be interpreted carefully since deviations from Benford's law distribution do not always indicate fraud in financial statements.

4. Empirical results

4.1. Research findings for the full sample

First, the full sample is tested for compliance with Benford's law without splitting it into control and fraud subsamples, as depicted in Figure 1. This allows the difference between the fraudulent and control firms to be shown in further analysis.

Figure 1. Distribution of the first, second, and first two digits in the full sample



Note: The theoretical values of Benford's law distribution for digits are the column values, and the actual distributions for the samples are indicated by the red points' values.

Source: Author's own estimation.

The results of the Benford analysis showed that 1s occurred more frequently than expected in the full sample, and there were too few 7s, 8s, and 9s (at the 10% significance level). These results indicated that companies could increase the value of their financial positions when the values were close to the 1s. Among the second digits, there were too few 9s and too many 1s, which could indicate that there were rounded-up multiples of 10. Also, the Z-statistics were exceeded more frequently for the first two digits ending in 9s, which is related to the second-digit distribution.

Table 3. MAD, SSD, BDS, Kolmogorov–Smirnov, and Kuiper test results for the full sample

Digits	MAD	SSD	BDS	Kolmogorov–Smirnov	Kuiper
First	0.0047***	2.445**		1.5689	2.328
Second	0.0041***	2.472**	0.0031***	1.2224*	2.434
First two	0.0010***	1.459***		1.8702	2.865

*** Indicates close conformity. ** Acceptable conformity. * Marginally acceptable conformity.

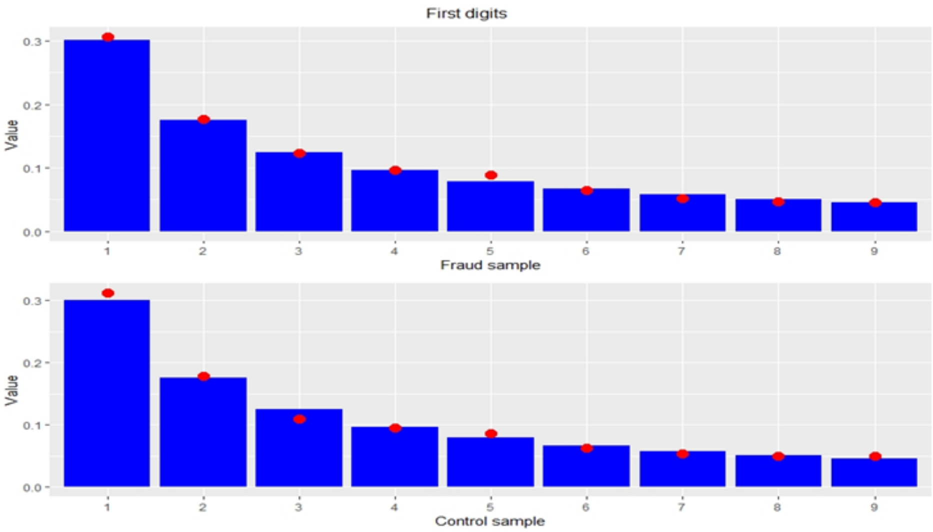
Source: Author's own estimation.

The results of diagnostic tests for the full sample are shown in Table 3. The results of the MAD and BDS tests show that the data have close conformity to Benford's distribution. However, the SSD test indicates that for the first and second digits, the data show acceptable conformity with Benford's law. However, the results from the KS and Kuiper tests demonstrate that the data do not follow Benford's distribution.

4.2. Research findings for fraud and control samples

Next, the sample was split into fraud and control subsamples for an analysis of the distribution of the digits, as depicted in Figure 2.

Figure 2. Distribution of first digits in fraud and control samples

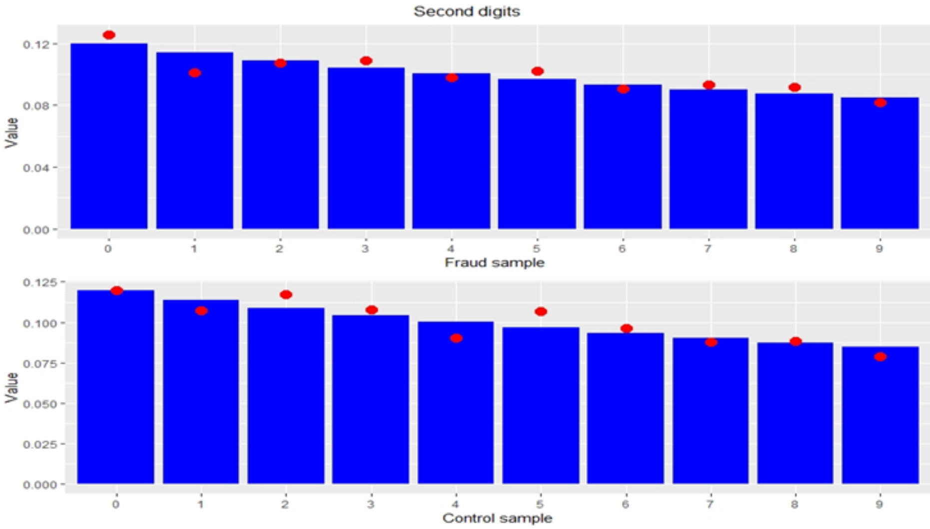


Note: The theoretical values of Benford's law distribution for digits are the column values, and the actual distributions for the samples are indicated by the red points' values.

Source: Author's own estimation.

Benford's analysis of the first digits indicates that 5s occur more frequently than expected and 7s occur less frequently in the fraud and control samples, but also for the control firms, 1s, and 9s occurred more frequently and 3s less frequently than expected, while for the fraudulent firms, 8s occurred less frequently than expected. The results also show that the distribution of the digits across the full sample was more influenced by the control firms' data than by the fraudulent firms' data.

Figure 3. Distribution of seconds digits in fraud and control samples



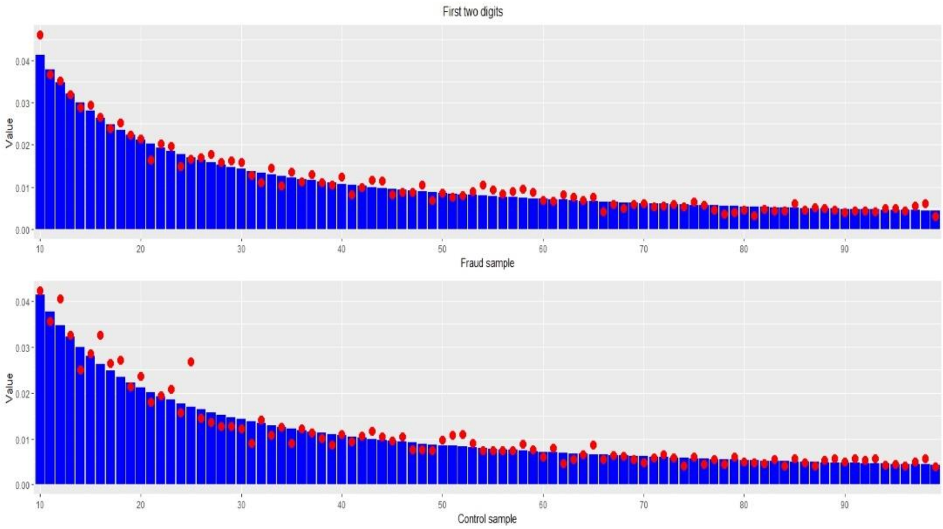
Note: The theoretical values of Benford's law distribution for digits are the column values, and the actual distributions for the samples are indicated by the red points' values.

Source: Author's own estimation.

As shown in Figure 3, among the second digits, there were too many 0s and too few 1s for the fraudulent firms. This result could indicate that there were rounded-up multiples of 10. For control firms, the Z-statistic values were significant for five digits, with fewer 1s and 4s than expected, in particular, and more 2s and 5s than expected. This can be caused by having too many numbers beginning with the larger digits due to having too few numbers beginning with the smaller digits. Also, the control firms' data have more influence on the full sample results than the fraudulent firms' data. However, after splitting the full sample into subsamples, the 3s follow the Benford distribution.

As shown in Figure 4, the Z-statistic value was exceeded by 22 digits for fraudulent firms and 25 digits for control firms. These results may indicate that there was no difference in the analysis of the distribution of the first two digits. However, the Z-statistic was exceeded more frequently for the first two digits ending in 9s for fraudulent firms, which is related to the second-digit distribution. Also, the fraudulent firms had more observations starting with 4 or 5 and fewer observations starting with 6 or 7, whereas the control firms had too many observations starting with 1 or 2 and too few observations starting with 3.

Figure 4. Distribution of first two digits in fraud and control samples



Note: The theoretical values of Benford's law distribution for digits are the column values, and the actual distributions for the samples are indicated by the red points' values.

Source: Author's own estimation.

Table 4. MAD, SSD, BDS, Kolmogorov–Smirnov, and Kuiper test results for fraud and control samples

Digits	MAD	SSD	BDS	Kolmogorov–Smirnov	Kuiper
Fraudulent observations (N = 14 863)					
First	0.0031***	1.723***		1.5859	3.937
Second	0.0043***	2.788**	0.0029***	1.0413**	2.585
First two	0.0010***	1.583***		1.8465	4.584
Control observations (N = 14 366)					
First	0.0055***	4.527**		1.5908	3.949
Second	0.0051***	3.771**	0.0040***	0.9269***	2.301
First two	0.0013**	3.668**		2.6072	6.473

*** Indicates close conformity. ** Acceptable conformity. * Marginally acceptable conformity.

Source: Author's own estimation.

The results of diagnostic tests for fraudulent and control observations are shown in Table 4. The results of the MAD and BDS tests show that the fraud and control samples have close conformity with Benford's distribution for the first and second digits. The main difference is in the first two digits results, where the control sample has acceptable conformity while the fraud sample has close conformity. This is not evident in Figure 4, where the fraud and control distributions look similar. Also, the MAD and BDS test results diverge less for the fraud sample than for the control and full samples, which suggests that the fraudulent firms show fewer deviations from the Benford distribution compared to the control firms. However, the results of the SSD test indicate that control firms have only acceptable conformity, while fraudulent companies have close conformity with Benford's law. Nevertheless, the results from the KS and Kuiper tests show that the data do not follow Benford's distribution, which indicates that these data are not suitable for applying Benford's law. Only for the second digits do the fraud and control sample data follow Benford's distribution.

5. Discussion

Earnings management is a global phenomenon that researchers are trying to detect using various techniques. Benford's law establishes that the frequencies of the appearance of digits in a number are fixed in a multitude of cases. The previous works show that Benford's law could be applied as a statistical audit tool to determine fraud and manipulations.

The results of this study show the close conformity of the full sample data for the first, second, and first two digits with Benford's distribution. The analyzed data satisfy the MAD, SSD, and BDS tests, indicating that it is appropriate to apply Benford's law. However, the results of the KS and Kuiper tests indicate that the data do not follow Benford's distribution. Also, the Z-statistic indicates deviations from Benford's law for specific digits: 5s and 9s for the second digits and 49s and 99s for the first two digits (Nigrini, 2017). The main findings show an inadequate distribution of financial numbers for the analyzed Polish companies. In addition, the BSD test has been adapted for the Turkish market; hence its effectiveness should be tested on other markets, and the necessary calibrations should be made. The application of tests to an observed accounting dataset cannot be considered to be conclusive, but it is one of several investigation tools that need to be utilized in detecting data errors (Kumar & Bhattacharya, 2007).

Similar results were obtained for the MAD, SSD, and BDS tests after splitting the sample into fraudulent and control companies. However, lower scores were obtained for fraudulent companies. Also, the Z-statistics indicated various digit deviations for the fraudulent and control companies. The Z-statistic value indicated more deviations in the distribution of digits for the control companies, and they were different from the fraud group. Nevertheless, the fraudulent companies' Z-statistics for 49s and 99s for the first two digits are below their critical values, and the Z-statistics for 0s and 5s for the second digits are above their critical values. This indicates that these values are important for the auditors, as they can reveal manipulation of data in the financial statements. Therefore, the results of previous research that did not distinguish between fraudulent and control firms may not have identified suitable areas for auditor analysis. KS and Kuiper's tests were used to check the actual proportions of the first, second, and first two digits in fraud and control samples and to determine whether they differ from Benford's law distribution. The tests show that, at a significance level of 5%, the discrepancy from Benford's law distributions does not exist for the financial statements of fraudulent and control companies. Based on the results achieved, the hypothesis was confirmed.

Additional tests were also performed for the four basic elements of financial statements (Tables 5 and 6).

Table 5. MAD, SSD, BDS, Kolmogorov–Smirnov, and Kuiper test results for basic elements of financial statements for fraud sample

Digits	MAD	SSD	BDS	Kolmogorov–Smirnov	Kuiper
Balance sheet (N = 4 912)					
First	0.0059***	6.762**	0.0047***	1.2033*	1.898
Second	0.0066***	6.270**		1.5385	2.498
First two	0.0017**	4.667**		1.2891*	2.382
Statement of income (N = 2 471)					
First	0.0048***	3.057**	0.0040***	1.0700**	1.330*
Second	0.0054***	4.717**		0.8479***	1.555*
First two	0.0018*	6.092**		1.3460*	2.180
Statement of cash flow (N = 4 138)					
First	0.0059***	4.723**	0.0040***	1.5704	1.951
Second	0.0045***	3.198**		0.3940***	0.868***
First two	0.0016**	3.789**		1.7251	2.178
Statement of changes in equity (N = 3 342)					
First	0.0057***	3.799**	0.0059***	0.5134***	1.134***
Second	0.0097**	13.277*		1.2338*	2.630
First two	0.0024	8.935**		0.6378***	1.311**

*** Indicates close conformity. ** Acceptable conformity. * Marginally acceptable conformity.

Source: Author's own estimation.

Table 6. MAD, SSD, BDS, Kolmogorov–Smirnov, and Kuiper test results for basic elements of financial statements for control sample

Digits	MAD	SSD	BDS	Kolmogorov–Smirnov	Kuiper
Balance sheet (N = 4 739)					
First	0.0069**	7.209**	0.0050***	1.2336*	1.824
Second	0.0063***	5.994**		1.0458**	2.326
First two	0.0017**	4.996**		1.6043	2.347
Statement of income (N = 2 589)					
First	0.0089**	9.788**	0.0059***	1.4864	2.303
Second	0.0067***	6.533**		1.1253**	1.718
First two	0.0020*	6.212**		1.6745	2.675
Statement of cash flow (N = 3 980)					
First	0.0056***	4.444**	0.0046***	1.4576	1.811
Second	0.0065***	6.282**		0.7654***	1.782
First two	0.0017**	4.646**		1.5939	2.470
Statement of changes in equity (N = 3 058)					
First	0.0124*	18.441*	0.0082***	1.5726	2.661
Second	0.0093**	15.377*		1.5470	3.007
First two	0.0030	20.618*		2.6042	4.231

*** Indicates close conformity. ** Acceptable conformity. * Marginally acceptable conformity.

Source: Author's own estimation.

The MAD test shows close conformity with Benford's law for the first and second digits and acceptable conformity for the first two digits for fraudulent companies, while for control companies, there was usually acceptable conformity for each element of the financial statement. Moreover, the SSD test for the Statement of Changes in Equity indicated acceptable conformity for fraudulent companies and only marginally acceptable conformity for control companies. Also, the KS test showed that only the Statement of Cash Flow data did not follow Benford's law for the fraud sample, while for the control sample, only the balance sheet data followed Benford's law. However, the Kuiper test also indicated nonconformity with Benford's distribution of balance sheet data for the fraud sample and all elements of financial statements for the control sample.

The results indicate that fraudulent companies pay more attention to the manipulation of financial data so that it is not easy for auditors to detect them, while the control companies have no reason to embellish the real data to conform with the Benford distribution.

6. Conclusions

This paper demonstrates the use of Benford's law to study the effectiveness of the detection of data manipulations. The assumed research hypothesis was verified. The hypothesis confirmed that there are differences in distributions of digits of figures reported in the financial statements of fraudulent and control observations.

The study contributes significantly to the new findings by demonstrating that the distribution of the digits between control and fraud companies can make a significant difference. The Z statistics often show other digits for samples whose frequencies deviate from Benford's law. Furthermore, statistical test results showed that fraud companies tend to have higher conformity with Benford's law than control companies and the full sample. The additional analysis performed on the four basic elements of financial statements revealed significant differences under Benford's law for control and fraud companies. Again, the data of the fraudulent companies showed much better compliance with Benford's law than the control companies. The study makes a significant theoretical and methodological contribution to the development of the literature on the detection of financial statement fraud using Benford's law. The previous studies have used Benford's law to analyze samples based on the selected positions of the financial statements of all companies, while earnings management detection studies divide the sample for fraud and control groups. The performing analyses in the distribution of the digits based on the mean results for all companies may lead to the exclusion of fraudulent companies from further verification, as their results are closer to the Benford distribution. These results have crucial implications for practice. The digits tests should be performed based on the mean values of the two types of companies: manipulators and non-manipulators. Otherwise, based on the average value of all companies, it will be much more difficult to detect manipulation of earnings management.

The findings should be considered in light of the following limitations. The study included only 63 control observations, which are the result of an analysis of the empirical literature, where one to three non-manipulators companies are typically selected for the sample. Therefore, the findings may be only applicable to the limited sample, so the analysis should be extended to all public companies during the research period. Including all public companies in the study will better determine the value of the Benford distribution conformity for the Polish market and provide a benchmark for the auditor's research.

Disclosure statement

No potential conflict of interest was reported by the author(s).

References

Alali, F., & Romero, S. (2013). Benford's Law: Analyzing a decade of financial data. *Journal of Emerging Technologies in Accounting, 10*(1), 1-39. https://doi.org/10.2308/jeta-50749

Amiram, D., Bozanic, Z., & Rouen, E. (2015). Financial statement errors: Evidence from the distributional properties of financial statement numbers. *Review of Accounting Studies, 20*, 1540-1593. https://doi.org/10.1007/s11142-015-9333-z

Asllani, A., & Naco, M. (2015). Using Benford's Law for fraud detection in accounting practices. *Journal of Social Science Studies, 2*(1), 129-143. https://doi.org/10.5296/jsss.v2i1.6395

Bader, A. A., & Saleh, M. M. A. (2017). Evidence on the extent of cosmetic earnings and revenues management by Jordanian companies. *International Journal of Economics and Financial Issues, 7*(3), 20-30. https://www.econjournals.com/index.php/ijefi/article/view/4370/pdf

Barney, B. J., & Schulzke, K. S. (2016). Moderating "cry wolf" events with excess MAD in Benford's Law research and practice. *Journal of Forensic Accounting Research, 1*(1), A66-A90. https://doi.org/10.2308/jfar-51622

Baryła, M. (2017). Analiza rozkładu pierwszej cyfry znaczącej danych finansowych wybranych spółek z sektora mediów notowanych na GPW w Warszawie [The first significant digit distribution analysis of financial data of selected companies from media sector listed on the Warsaw Stock Exchange]. *Prace Naukowe Uniwersytetu Ekonomicznego we Wrocławiu=Research Papers of Wrocław University of Economics, 469*, 11-20. https://doi.org/10.15611/pn.2017.469.01

Benford, F. (1938). The law of anomalous numbers. *Proceedings of the American Philosophical Society, 78*(4), 551-572. https://www.jstor.org/stable/984802

Berger, A., & Hill, T. P. (2015). *An introduction to Benford's law*. New Jersey: Princeton University Press. https://doi.org/10.23943/princeton/9780691163062.001.0001

Carslaw, C. A. P. N. (1988). Anomalies in income numbers: Evidence of goal oriented behavior. *Accounting Review, 68*(2), 321-327. https://www.jstor.org/stable/248109

Davis, C. S., & Stephens, M. A. (1989). Algorithm AS 248: Empirical distribution function goodness-of-fit tests. *Applied Statistics, 38*(3), 535-582. https://doi.org/10.2307/2347751

Diaconis, P., & Freedman, D. (1979). On rounding percentages. *Journal of the American Statistical Association, 74*(366a), 359-364. https://doi.org/10.1080/01621459.1979.10482518

Drake, P. D., & Nigrini, M. J. (2000). Computer assisted analytical procedures using Benford's Law. *Journal of Accounting Education, 18*(2), 127-146. https://doi.org/10.1016/S0748-5751(00)00008-7

Durtschi, C., Hillison, W., & Pacini, C. (2004). The effective use of Benford's law to assist in detecting fraud in accounting data. *Journal of Forensic Accounting, 5*(1), 17-34. http://lycofs01.lycoming.edu/~sprgene/M400/BenfordsLaw.pdf

Guan, L., He, D., & Yang, D. (2006). Auditing, integral approach to quarterly reporting, and cosmetic earnings management. *Managerial Auditing Journal, 21*(6), 569-581. https://doi.org/10.1108/02686900610674861

Hales, D. N., Sridharan, V., Radhakrishnan, A., Chakravorty, S. S., & Siha, S. M. (2008). Testing the accuracy of employee-reported data: An inexpensive alternative approach to traditional methods. *European Journal of Operational Research, 189*(3), 583-593. https://doi.org/10.1016/j.ejor.2006.09.092

Hill, T. P. (1995). A statistical derivation of the significant-digit law. *Statistical Science, 10*(4), 354-363. https://doi.org/10.1214/ss/1177009869

Istrate, C. (2019). Detecting earnings management using Benford's Law: The case of Romanian listed companies. *Journal of Accounting and Management Information Systems, 18*(2), 198-223. https://doi.org/10.24818/jamis.2019.02003

Johnson, G. C. (2009). Using Benford's law to determine if selected company characteristics are red flags for earnings management. *Journal of Forensic Studies in Accounting & Business, 1*(2), 39-65.

Jordan, C. E., Clark, S. J., & Hames, C. (2009). Manipulating sales revenue to achieve cognitive reference points: An examination of large U.S. public companies. *Journal of Applied Business Research, 25*(2), 95-103. https://doi.org/10.19030/jabr.v25i2.1039

Kossovsky, A. E. (2014). *Benford's Law: Theory, the general law of relative quantities, and forensic fraud detection applications*. New Jersey: World Scientific.

Kossovsky, A. E. (2021). On the mistaken use of the chi-square test in Benford's Law. *Stats, 4*(2), 419-453. https://doi.org/10.3390/stats4020027

Kuiper, N. H. (1960). Tests concerning random points on a circle. *Indagationes Mathematicae (Proceedings), 63*, 38-47. https://doi.org/10.1016/S1385-7258(60)50006-0

Kumar, K., & Bhattacharya, S. (2007). Detecting the dubious digits: Benford's law in forensic accounting. *Significance, 4*(2), 81-83. https://doi.org/10.1111/j.1740-9713.2007.00234.x

Kumar, S. B., Goyal, V., & Mitra, S. K. (2018). Do Indian firms manage earning numbers? An empirical investigation. *Academy of Accounting and Financial Studies Journal, 22*(1), 1-7. https://www.abacademies.org/articles/Do-indian-firms-manage-earning-numbers-an-empirical-investigation-1528-2635-22-1-113.pdf

Mataković, I. C. (2019). The empirical analysis of financial reports of companies in Croatia: Benford distribution curve as a benchmark for first digits. *Croatian Review of Economic, Business and Social Statistics, 5*(2), 90-100. https://doi.org/10.2478/crebss-2019-0014

Máté, D., Sadaf, R., Tarnóczi, T., & Fenyves, V. (2017). Fraud detection by testing the conformity to Benford's law in the case of wholesale enterprises. *Polish Journal of Management Studies, 16*(1), 115-126. https://doi.org/10.17512/pjms.2017.16.1.10

Miller, S. J., & Nigrini, M. J. (2008). Order statistics and Benford's law. *International Journal of Mathematics and Mathematical Sciences, 2008*, ID 382948, 1-19. https://doi.org/10.1155/2008/382948

Morales, H. R., Porporato, M., & Epelbaum, N. (2022). Benford's law for integrity tests of high-volume databases: A case study of internal audit in a state-owned enterprise. *Journal of Economics, Finance and Administrative Science, 27*(53), 154-174. https://doi.org/10.1108/JEFAS-07-2021-0113

Morrow, J. (2014). *Benford's Law, families of distributions and a test basis* (Discussion Paper, No. 1291). Centre for Economic Performance, London School of Economics and Political Science. http://eprints.lse.ac.uk/60364/1/dp1291.pdf

Nigrini, M. J. (1994). Using digital frequencies to detect fraud. *The White Paper, 8*(2), 3-6.

Nigrini, M. J. (1996). A taxpayer compliance application of Benford's Law. *The Journal of the American Taxation Association, 18*(1), 72-91. https://www.proquest.com/docview/211023799/fulltextPDF/424F86C4DACC4784PQ/1?accountid=45580

Nigrini, M. J. (2000). *Digital analysis using Benford's law: Tests and statistics for auditors*. Vancouver: Global Audit Publications. https://doi.org/10.1201/1079/43266.28.9.20010301/30389.4

Nigrini, M. J. (2005). An assessment of the change in the incidence of earnings management around the Enron-Andersen episode. *Review of Accounting and Finance, 4*(1), 92-110. https://doi.org/10.1108/eb043420

Nigrini, M. J. (2015). Persistent patterns in stock returns, stock volumes, and accounting data in the US capital markets. *Journal of Accounting, Auditing & Finance, 30*(4), 541-557. https://doi.org/10.1108/eb043420

Nigrini, M. J. (2017). Audit sampling using Benford's law: A review of the literature with some new perspectives. *Journal of Emerging Technologies in Accounting, 14*(2), 29-46. https://doi.org/10.2308/jeta-51783

Nigrini, M. J. (2020). *Forensic analytics: Methods and techniques for forensic accounting investigations*. New Jersey: John Wiley & Sons.

Nigrini, M. J. (2022). Using Benford's Law to reveal journal entry irregularities: Benford's Law can help uncover indicators of fraud and anomalies that arise from legitimate business practices. *Journal of Accountancy, 234*(3), 12-20. https://www.journalofaccountancy.com/issues/2022/sep/using-benfords-law-reveal-journal-entry-irregularities.html

Nigrini, M. J., & Mittermaier, L. J. (1997). The use of Benford's Law as an aid in analytical procedures. *Auditing, 16*(2), 52-67. https://www.thefreelibrary.com/The+use+of+Benford%27s+Law+as+an+aid+in+analytical+procedures-a020746462

Özarı, C., & Ocak, M. (2013). Detection of earnings management by applying Benford's Law in selected accounts: Evidence from quarterly financial statements of Turkish public companies. *European Journal of Economics, Finance and Administrative*

Sciences, 59(4), 37-52. https://www.researchgate.net/publication/270278179_Detection_of_Earnings_Management_by_Applying_Benford%27s_Law_in_Selected_Accounts_Evidence_From_Quarterly_Financial_Statements_of_Turkish_Public_Companies

Özevin, O., Yücel, R., & Öncü, M. A. (2020). Fraud detecting with Benford's law: An alternative approach with BDS and critic values. *Muhasebe Bilim Dünyası Dergisi, 22*(1), 107-126. https://doi.org/10.31460/mbdd.609957

Roxas, M. L. (2011). Financial statement fraud detection using ratio and digital analysis. *Journal of Leadership, Accountability, and Ethics, 8*(4), 56-66. http://www.na-businesspress.com/JLAE/Roxas84Web.pdf

Saville, A. D. (2006). Using Benford's Law to detect data error and fraud: An examination of companies listed on the Johannesburg Stock Exchange. *South African Journal of Economic and Management Sciences, 9*(3), 341-354. https://doi.org/10.4102/sajems.v9i3.1092

Skousen, C. J., Guan, L., & Wetzel, T. S. (2004). Anomalies and unusual patterns in reported earnings: Japanese managers round earnings. *Journal of International Financial Management & Accounting, 15*(3), 212-234. https://doi.org/10.1111/j.1467-646X.2004.00108.x

Tam Cho, W. K., & Gaines, B. J. (2007). Breaking the (Benford) Law: Statistical fraud detection in campaign finance. *The American Statistician, 61*(3), 218-223. https://doi.org/10.1198/000313007X223496

Tilden, C., & Janes, T. (2012). Empirical evidence of financial statement manipulation during economic recessions. *Journal of Finance and Accountancy, 10*(1), 1-15. https://www.aabri.com/OC2012Manuscripts/OC12064.pdf

Tsagbey, S., de Carvalho, M., & Page, G. L. (2017). All data are wrong, but some are useful? Advocating the need for data auditing. *The American Statistician, 71*(3), 231-235. https://doi.org/10.1080/00031305.2017.1311282

Zdraveski, D., & Janeska, M. (2021). Application of Benford's law for detecting manipulation in the financial statements in Macedonian companies. *Annals of the "Constantin Brancusi" University of Targu Jiu, Economy Series, 6*, 4-13. https://www.utgjiu.ro/revista/ec/pdf/2021-06/01_Dejan.pdf

Žgela, M., & Dobša, J. (2011). Analysis of top 500 Central and East European companies net income using Benford's law. *Journal of Information and Organizational Sciences, 35*(2), 215-228. https://jios.foi.hr/index.php/jios/article/view/205