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IMPLICATIONS OF PARAMETER SELECTION IN DYNAMIC MULTIOBJECTIVE MODELS IN ECONOMICS AND FINANCE

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Abstract

Time is a key variable in the field of economics and finance. However, most of the classic approaches to economic problems are static. In this paper, we first review the existing literature on the use of multiobjective techniques to control dynamic systems within the area of economics and finance. We also tackle the question of which measure should we use to evaluate alternative solutions. To this end, we elaborate on the meaning added by the selection of a parameter in a family of distance functions used to evaluate alternative solutions.

Keywords: time, dynamic systems, economics, distance function, review.

1 Introduction

Multiple criteria decision making (MCDM) problems are characterized by the presence of several conflicting objectives that are considered simultaneously. We

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formulate each relevant aspect as an objective function and we use multiobjective optimization to find the best solutions. MCDM dates back to the works by Pareto at the end of the 19th century, but the field has grown very fast during the last decades. Some general works on MCDM include, but are not limited to, Yu (1985), Steuer (1986), Romero (1991), Ballestero and Romero (1998), Erhgott (2005), Jones and Tamiz (2010), and Zopounidis and Pardalos (eds., 2010).

Within the context of MCDM, there is a group of problems in which time is a key variable in addition to the criteria under consideration. Time is particularly relevant in the field of economics and finance. However, most of the classic approaches to economic problems are static as in Ballestero and Romero (1998). In this paper, we focus on multiobjective techniques used to control dynamic systems. By dynamic multiobjective problems we mean multiperiod problems in which we want to optimize a set of objective functions over time. This definition includes problems in which we want to optimize the final or cumulative state of criteria, as in Caballero et al. (1998), but also the deviation of the trajectory of these criteria over time with respect to a given reference as described in Wierzbicki (1988). As a result, the first goal of this paper is to review relevant papers in multiobjective control within the fields of economics and finance. We restrict ourselves to economic and financial models because time series and multistage problems are ubiquitous in these areas.

An important research question arises when dealing with multiobjective control problems: which measure should we use to evaluate alternative solutions? If we adhere to dynamic goal programming, we should minimize the sum of deviations for each time step. However, other alternatives suggest the use of maximum absolute deviations or percentage deviations. Most of the alternatives are indeed special cases of the Minkowski distance function when a parameter is set to a particular integer value. As a second goal of the paper, we discuss the implications of selecting this parameter, extending the work by Gonzalez-Pachon and Romero (2016) in terms of the meaning added to the process. More precisely, we argue that there is an implicit selection of a decision-making principle when this parameter is set to a value. Finally, we reformulate multiobjective control problem as a constrained norm approximation problem. This reformulation presents the advantage of being convex and then solvable for any order of the norm (value of the parameter) using state-of-the-art convex optimization algorithms.

Summarizing, the contributions of this paper are twofold:

1. A review of relevant papers in multiobjective dynamic models in economics and finance.
2. An analysis of the decision-making principles that underlie the selection of a parameter in the Minkowski distance function.

In addition to this introduction, the structure of this paper is as follows. Section 2 reviews a set of the most relevant papers in multiobjective dynamic control. Section 3 discusses the implications of selecting a parameter in multi-objective dynamic control problems. Section 4 provides concluding remarks.

2 A review of multiobjective dynamic models in economics and finance

We can set the origins of a formal treatment of dynamic systems in the book by Bellman (1957). In this book, Bellman provided an introduction to the mathematical theory of multistage decision processes and introduced the notion of dynamic programming (DP) to describe the subject matter. Other basic concepts, such as the Bellman equation to derive optimal policies in a recursive manner, paved the way to recent advances in economics. For instance, Sargent and Ljungqvist (2000) used several recursive methods to study macroeconomics while Hansen and Sargent (2013) studied recursive models of dynamic linear economies. Briefly, recursive models break a multistage problem into small pieces by forming a sequence of time-dependent problems.

Following the definition by Kall and Wallace (1994), dynamic problems are characterized by stages or time steps indexed by t, the state x_t at time t, the decision taken u_t at time t, the transformation of the system from the current state and the decision taken to the next state, the return r_t (x_t, u_t) obtained at time t, the set X of feasible decisions, and the overall objective function F which depends on the returns r_t for the whole planning horizon T. The consideration of time as a key variable in a decision-making problem adds a new level of complexity to the problem. Indeed, Bellman (1957) refers to dynamic problems as multidimensional maximization problems. In this paper, we argue that a natural way to deal with multidimensional problems is to use multiple criteria decision making (MCDM).

MCDM covers a wide range of techniques as described in Yu (1985), Steuer (1986), Romero (1991), Ballestero and Romero (1998), Erhgott (2005), Jones and Tamiz (2010), and Zopounidis and Pardalos (eds., 2010). Among them, Goal Programming (GP) initially proposed by Charnes and Cooper (1957) is one of the most widely used techniques. The introduction of dynamic features into the problem led to the development of Dynamic Goal Programming (DGP). Yu and Leitmann (1974) considered a dynamic multiobjective decision problem in which the concepts of non-dominated solutions were extended to a dynamic context. The use of trajectories over the planning horizon that play the role of a reference signal for optimization purposes was proposed by Wierzbicki (1980; 1988).

Daellenbach and de Kluyver (1980) introduced a multiobjective dynamic programming (MODP) technique as an extension of dynamic programming concepts. Levary (1984) proposed a scalarization approach by means of GP. Later on, Li and Haimes (1989) highlighted the development of the research area and reviewed both the concepts and the works in relation to theory and practice of MODP. On the other hand, Opricovic (1993) developed a compromise programming method (Zeleny, 1973) by minimizing the distance to the ideal solution within a dynamic context and with application to water reservoir management.

In the proceedings of two multiple criteria decision-making conferences, Trzaskalik (1997a; 1997b) discussed several aspects, such as monotonicity and separability in a multiple criteria context. Caballero et al. (1998) described an approach with dynamic target values to control not only the final values of the objective functions, but also their evolution along the planning horizon. Discrete dynamic programming with partially ordered criteria set was also considered by Trzaskalik and Sitarz (2002; 2007). More recently, Trzaskalik (2022) provided novel theoretical results on the possibility of finding the best multistage policies using Bellman's optimality principle and the multicriteria bipolar method in which two sets of references points are determined.

Zopounidis and Doumpos (2013) analyzed the importance of multicriteria decision systems for financial problems and reviewed the most relevant papers in two main areas of financial decision support, namely, portfolio selection and corporate performance evaluation. In what follows, we adopt a similar approach, but focusing only on the dynamic aspects of multicriteria decision-making models in finance.

In what follows, we pay special attention to the dynamic portfolio selection problem. Probably the most studied problem in multicriteria financial decision-making is the portfolio selection problem, due to the pioneering work by Markowitz (1952). However, the classical mean-variance model by Markowitz (1952) is a one-period model. This fact is critical because investors are usually concerned with cumulative results over a period of time and optimal decisions for a single period may be suboptimal in a multiperiod framework according to Estrada (2010). To overcome this limitation, Kelly (1956) in the context of gambling and Latane (1959) in the context of investing proposed a multiperiod framework with cumulative results which is equivalent to the maximization of the geometric mean of returns.

Mossin (1968) proposed an extension of the one-period model to a multiperiod framework following a dynamic programming approach and acknowledging first that Tobin (1965) appeared to be one of the first authors to make an attempt in this direction. What is most interesting in Mossin (1968) is the definition of the single-period problem and the multiperiod problem that we reproduce here for clarity:

> "By a single-period model is meant a theory of the following structure: The investor makes his portfolio decision at the beginning of a period and then waits until the end of the period when the rate of return on his portfolio materializes. He cannot make any intermediate changes in the composition of his portfolio. The investor makes his decision with the objective of maximizing expected utility of wealth at the end of the period (final wealth)" (Mossin, 1968, p. 216).

> "By a multiperiod model is meant a theory of the following structure: The investor has determined a certain future point in time (his horizon) at which he plans to consume whatever wealth he has then available. He will still make his investment decisions with the objective of maximizing expected utility of wealth at that time. However, it is now assumed that the time between the present and his horizon can be subdivided into *n* periods (not necessarily of the same length), at the end of each of which return on the portfolio held during the period materializes and he can make a new decision on the composition of the portfolio to be held during the next period" (Mossin, 1968, p. 220).

Instead of maximizing expected utility functions of the terminal wealth and/or multiperiod consumption, Li and Ng (2000) proposed an analytical method for the mean-variance formulation to find the multiperiod optimal portfolio policy. Zhou and Li (2000) also used the mean-variance formulation to select portfolios in a continuous framework using a stochastic linear-quadratic model. This line of work was later extended by Basak and Chabakauri (2010), Wang and Zhou (2020), Dai et al. (2021) and many others.

More recently, Ben Abdelazziz et al. (2020) also proposed a stochastic dynamic multiobjective model for sustainable decision-making with applications in sustainable portfolio management with two stocks and two criteria (return and sustainability), and also in a workforce allocation problem in an economy with two sectors.

A novel line of research has recently arised from the application of the multi-objective dynamic techniques derived from the portfolio selection problem to the cash management problem by Salas-Molina, Pla-Santamaria and Rodriguez--Aguilar (2018a), Salas-Molina, Pla-Santamaria and Rodríguez-Aguilar (2018b), Salas-Molina, Rodríguez-Aguilar and Pla-Santamaria (2018) and Salas-Molina (2019). In this area of research, Sethi and Thomson (1970; 2000) proposed an optimal control theory approach to the cash management problem that has been recently extended by Bhaya and Kaszkurewicz (2022) in a single-objective context.

3 Implications of parameter selection in multiobjective dynamic control models

The main goal of this section is to provide a way to add meaning to multiobjective dynamic models by selecting a particular form of the objective function used for optimization purposes. To this end, we first formulate a general dynamic goal program in which a parametric distance function is used to find the best solutions. Second, we analyze the implications of selecting a key parameter in this distance function in terms of the implicit decision-making principle derived from this choice. We illustrate the implications by means of the analysis of the most important cases.

Let us start with the classical GP formulation by Charnes and Cooper (1957):

$$min\ \sum_{i=1}^{q}(w_i^+\delta_i^+ + w_i^-\delta_i^-) \tag{1}$$

subject to:

$$g_i(\boldsymbol{u},\boldsymbol{x}) + \delta_i^- - \delta_i^+ = b_i \tag{2}$$

$$\delta_i^-, \delta_i^+ \geq 0 \tag{3}$$

$$u \in S \tag{4}$$

where we consider the positive $\boldsymbol{\delta_i^+}$ and negative deviations $\boldsymbol{\delta_i^-}$ of q different goals achievements measured by $\boldsymbol{g_i}(\boldsymbol{u},\boldsymbol{x})$ from targets $\boldsymbol{b_i}$. Goal achievements depend on control actions in vector $\boldsymbol{u}$ subject to some feasibility set S and states in vector $\boldsymbol{x}$.

By including time as a key variable in the previous GP formulation, we are dealing with a multiobjective control problem described as the minimization of deviations with respect to some dynamic targets or trajectories as proposed, for instance, by Wierzbicki (1988) and Caballero et al. (1998). We are dealing with a dynamic goal program (DGP):

$$\min \sum_{i=1}^{q}\sum_{t=1}^{n}(w_i^+\delta_{it}^+ + w_i^-\delta_{it}^-) \tag{5}$$

subject to:

$$g_{it}(\boldsymbol{u},\boldsymbol{x}) + \delta_{it}^- - \delta_{it}^+ = b_{it} \tag{6}$$

$$\delta_{it}^-, \delta_{it}^+ \geq 0 \tag{7}$$

$$\boldsymbol{u} \in S \tag{8}$$

We can now move one step further by considering parameter p in the DGP formulation:

$$\min \left[\sum_{\mathrm{i}=1}^{\mathrm{q}} \sum_{t=1}^{n} (w_i^+ \delta_{it}^+)^p + (w_i^- \delta_{it}^-)^p\right]^{1/\mathrm{p}} \tag{9}$$

subject again to equations (6), (7) and (8). The use of this parameter allows us to increase the degree of generality and, at the same time, to add meaning to the optimization process. We increase the degree of generality because we are able to consider not only linear deviations but also quadratic or maximum deviations as in the case of the Chebyshev variant of the classical linear GP formulation. Furtheremore, we are adding meaning to the optimization process, because by setting p, we are implicitly selecting a decision-making principle as we elaborate it next.

3.1 Case p = 1, linear control and the principle of maximum efficiency

For simplicity of notation, let us assume that all goal functions $\boldsymbol{g_i}(\boldsymbol{u}, \boldsymbol{x})$ are equally-weighted normalised non-negative linear functions of states in vector $\boldsymbol{x}$ and controls in vector $\boldsymbol{u}$ that are subject to a given set of constraints. As a result, when we set $\boldsymbol{p} = \mathbf{1}$ in equation (9), we are indeed minimizing the sum of absolute deviations. And this minimization can be viewed as the application of the principle of maximum efficiency (Gonzalez-Pachon and Romero, 2016), because we focus on the sum of achievements disregarding particular deviations in favour of the sum (majority) of deviations.

$$\min \sum_{i=1}^{q} \sum_{t=1}^{n} |g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it}| \tag{10}$$

In this case, we apply a multiobjective linear control of a set of q goals determined by a set of dynamic targets (or trajectories) over a planning horizon of n time steps.

3.2 Case p = ∞, minimax control and the principle of maximum fairness

Now consider the case when we set $\boldsymbol{p} = \infty$ in equation (9). In this case, we minimize the maximum absolute deviations and this minimization can be viewed as the application of the principle of maximum fairness, because we focus on the worst observation as suggested by Gonzalez-Pachon and Romero (2016).

$$\min \left[\sum_{i=1}^{q} \sum_{t=1}^{n} |g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it}|^{\infty}\right]^{1/\infty} \rightarrow \min \max(|g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it}|) \tag{11}$$

As a result, we apply a minimax control of a set of q goals determined by a set of dynamic targets (or trajectories) over a planning horizon of n time steps. The main implication here is that we are making decisions based on a single observation and this could be a problem for long time horizons.

3.3 Case p = 2, quadratic control and the principle of balance

When $\boldsymbol{p} = \mathbf{2}$, we minimize the Euclidean distance between a reference signal (dynamic targets) and goal achievement. This minimization can be viewed as the application of the principle of balance because we are somewhere in between the cases $\boldsymbol{p} = \mathbf{1}$ and $\boldsymbol{p} = \infty$.

$$\min \left[\sum_{i=1}^{q} \sum_{t=1}^{n} (g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it})^2\right]^{1/2} \tag{12}$$

In this case, we apply a quadratic control of a set of q goals determined by a set of dynamic targets (or trajectories) over a planning horizon of n time steps. This approach represents a compromise between the principle of maximum efficiency when $\boldsymbol{p} = \mathbf{1}$ (the rule of the majority) and the principle of maximum fairness when $\boldsymbol{p} = \infty$ (the rule of the most disadvantaged).

3.4 Case p = 0, geometric control and the principle of limited compensability

There is another case which is not so common in the literature but which leads to another important decision-making principle, namely, the principle of limited compensability. It can be shown that when $\boldsymbol{p} = \mathbf{0}$, equation (9) is equivalent to considering the product of deviations. This approach implies the principle of limited compensability because we limit the offset between bad performance in one deviation with superior performance in other deviations:

$$\left[\sum_{i=1}^{q} \sum_{t=1}^{n} |g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it}|^{p}\right]^{1/\mathrm{p}} \rightarrow \prod_{i=1}^{q} \prod_{t=1}^{n} |g_{it}(\boldsymbol{u}, \boldsymbol{x}) - b_{it}| \tag{13}$$

As a result, we apply a geometric control of a set of q goals determined by a set of dynamic targets (or trajectories) over a planning horizon of n time steps. In this case, it is convenient to set the targets to the anti-ideal values and maximize the product of deviations with respect to these anti-ideal values, because otherwise, a single null deviation would lead to a minimum of the functional.

3.5 Combination of decision-making principles

As suggested by Gonzalez-Pachon and Romero (2016), we can use parameter $\boldsymbol{\lambda}$ to produce a combination of decision-making principles. For instance, we can consider a weighted combination of the principle of maximum efficiency and the principle of maximum fairness by minimizing the following functional:

$$\lambda \mathcal{L}_1 + (1-\lambda)\mathcal{L}_\infty \tag{14}$$

where $\boldsymbol{\mathcal{L}_1}$ and $\boldsymbol{\mathcal{L}_\infty}$ are, respectively, the parametric distance function in equation (9) when $\boldsymbol{p = 1}$ and $\boldsymbol{p} = \infty$. We can extend this approach by considering any value of p in the range between zero and infinity as a potential representation of an additional decision-making principle. For instance, $\boldsymbol{p = 3}$ can be viewed, at least in theory, as a new principle that lies between the principle of maximum efficiency and the principle of maximum fairness and one step further of the principle of balance represented by $\boldsymbol{p = 2}$.

3.6 Solving the problem

It is obvious that when $p > 1$, we are dealing with a non-linear problem that may result in difficulties to find the optimal policies. However, the minimization of a sum of deviations of q goals over planning horizon n raised to p and the whole sum raised to *1/p* is equivalent to the minimization of the *p*-norm of a vector of dimension $n \cdot q$:

$$\min \left\| [\delta_{11}, \delta_{12}, \dots, \delta_{nq}] \right\|_p \tag{15}$$

If the minimization of the *p*-norm of a vector of deviations with respect to dynamic targets is subject to a set of linear constraints, we are dealing with a constrained norm approximation problem. Fortunately, this problem is convex and can be solved for any value of p using state-of-the-art convex optimization algorithms such as CVXPY within CPLEX or Gurobi (Boyd and Vandenberghe, 2004). When we consider non-linear goals or constraints, we need to apply some heuristics to solve the problem.

4 Concluding remarks

The main research question addressed in this paper is whether we can add meaning to the optimization process in multiobjective control. To this end, we consider dynamic goal programming that usually deals with linear-quadratic control problems as a starting point to propose a more general approach. This approach is based on the selection of a parameter of the Minkowski distance function. Extending previous works on the subject, we show that the selection of this parameter implies the use of multiple decision-making principles that may help practitioners to motivate the use of objective functions to derive control policies.

We also highlight the point that any value of p can be interpreted as representative of any decision-making principle. In order to deal with non-linearity of some of the decision-making principles, we suggest the use of constrained norm approximation methods to solve a general multiobjective control problem.

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