



Kateryna Czerniachowska

<https://orcid.org/0000-0002-1808-6020>

Department of Process Management
Faculty of Management
Wroclaw University of Economics
and Business, Poland
kateryna.czerniachowska@ue.wroc.pl

Krzysztof Lutosławski

<https://orcid.org/0000-0002-2978-1184>

Department of Process Management
Faculty of Management
Wroclaw University of Economics
and Business, Poland
krzysztof.lutoslawski@ue.wroc.pl

Marcin Hernes

<https://orcid.org/0000-0002-3832-8154>

Department of Process Management
Faculty of Management
Wroclaw University of Economics
and Business, Poland
marcin.hernes@ue.wroc.pl

Linear and nonlinear shelf space allocation problems with vertical and horizontal bands

Accepted by Editor Ewa Ziemba | Received: February 14, 2022 | Revised: April 20, 2022 | Accepted: May 7, 2022 | Published: May 23, 2022.

© 2022 Author(s). This article is licensed under the Creative Commons Attribution-NonCommercial 4.0 license (<https://creativecommons.org/licenses/by-nc/4.0/>)

Abstract

Aim/purpose – Shelf space is one of the most important tools for attracting customers' attention in a retail store. This paper aims to develop a practical shelf space allocation model with visible vertical and horizontal categories. and formulate it in linear and non-linear forms.

Design/methodology/approach – The research is mainly based on operational research. Simulation, mathematical optimization, and linear and nonlinear programming methods are mainly used. Special attention is given to the decision variables and constraints. Changing the dimensioning of the decision variables results in an improvement in the formulation of the problem, which in turn allows for obtaining an optimal solution.

Findings – A comparison of the developed shelf space allocation models with visible vertical and horizontal categories in linear and nonlinear forms is presented. The computational experiments were performed with the help of CPLEX solver, which shows that

Cite as: Czerniachowska, K., Lutosławski, K., & Hernes M. (2022). Linear and non-linear shelf space allocation problems with vertical and horizontal bands. *Journal of Economics & Management*, 44, 119-141. <https://doi.org/10.22367/jem.2022.44.06>

the optimal solution of the linear problem formulation was obtained within a couple of seconds. However, a nonlinear form of this problem found the optimal solution only in 19 out of 45 instances. An increase in the time limits slightly improves the performance of the solutions of the nonlinear form.

Research implications/limitations – The main implication of research results for science is related to the possibility of determining an optimal solution to the initially formulated nonlinear shelf space allocation problem. The main implication for practice is to take into consideration the practical constraints based on customers' requirements. The main limitations are the lack of storage conditions and holding time constraints.

Originality/value/contribution – The main contribution is related to developing mathematical models that consider simultaneous categorization of products vertically, based on one characteristic, and horizontally, based on another characteristic. Contribution is also related to extending the shelf space allocation theory with the shelf space allocation problem model in relation to four sets of constraints: shelf constraints, product constraints, orientation constraints, and band constraints.

Keywords: Retailing, decision making/process, merchandising, shelf space allocation, planogram.

JEL Classification: C61, L81.

1. Introduction

One of the most valuable resources in the retail sector is shelf space (Hwang, Choi, & Lee, 2009). As a result, the present shelf space management decision is a critical issue in retail operations. Retailers gain from the optimal product assignment on shelves in two different ways: they save money on shelf replacement and stock, and they increase sales.

The shelf space allocation problem (SSAP) is utilized in retail stores as a decision problem to achieve the maximum possible profit while working within operational constraints. In principle, commercial space management systems build operational procedures based on very simple intuitive guidelines that make it easy to decide about shelf space allocation in practice (Akkaş, 2019; Yang & Chen, 1999).

The visual characteristics of the assortment impact reflexive customers' attention. The relative visibility of items within the assortment, the position factors of those products on display, the number of facings, and the display size are all examples of these characteristics. Color block, for example, can be used by retailers to draw additional spontaneous attention (Kahn, 2017).

The decision to make workload can also be reduced through categorization, assortment management hierarchy, grouping, and other merchandising rules (Kahn, 2017). The existing research (for instance, Bianchi-Aguiar, Silva, Guimarães, Car-

ravilla, & Oliveira, 2018; Düsterhöft, Hübner, & Schaal, 2020), however, takes into account horizontal and vertical rules separately. There is a lack of simultaneous product categorization on horizontal and vertical rules, which beautifies the planogram, and allows easier product understanding and comparison. Constraints that take greater account of customer requirements should also be factored in (Ghazavi & Lotfi, 2016). Therefore, the very important problem is to define such categorization possibilities, specify grouping parameter values, and include them in a novel SSAP.

This paper aims to develop a shelf space allocation model with visible vertical and horizontal categories. The model is based on linear and nonlinear programming. We used the same criteria function of planogram profit maximization as Hansen, Raut, and Swami (2010). The main formula repeats the allocation for each product on each shelf to sum the total profit across all products on all shelves of a planogram. The main contribution relies on developing mathematical models that consider simultaneous categorization of products vertically, based on one characteristic (e.g., type, brand, color), and horizontally, based on another characteristic (e.g., package type). These vertical and horizontal categories form visible bands on a planogram. To the best of our knowledge, such a model has not yet been developed.

Contribution is also related to extending the shelf space allocation theory with the SSAP model concerning new sets of constraints including the customer requirements in a deeper way: shelf constraints, product constraints, orientation constraints, and band constraints.

The rest of the paper is organized as follows. Section 2 presents the related works on shelf space and visual merchandising, focusing on product categorization. Section 3 provides the problem definition and mathematical models for both options. Computational experiments are executed in Section 4. Next, in Section 5, the discussion is presented. Conclusions are drawn in Section 6.

2. Literature review

2.1. Shelf space allocation

The task of efficiently placing products on shelves to maximize profit, enhance stock control, and improve customer pleasure, among other things, is known as shelf space allocation. If retailers want to keep innovating, they need a competitive advantage. Capturing a larger market share and increasing sales is

a strategy to accomplish this goal. Improving retail operations is one of the approaches to boost sales. Making efficient use of the limited shelf space in stores is a key tactic (Landa-Silva, Marikar, & Le, 2009).

SSAP is currently the subject of a lot of research. Customers can easily identify, compare, and approach the products sold at the store, thanks to appropriate shelf management selections (Borin, Farris, & Freeland, 1994).

Product demand is impacted by price, promotion, and assortment variety in the broader retail environment, as well as own-label programs and advertising (Dhar, Hoch, & Kumar, 2001). Lim, Rodrigues, and Zhang (2004) provided an extended model that took into account the influence of product groupings, such as putting products from the same category together or apart, as well as the impact of nonlinear profit functions. In most cases, category and shelf space considerations should take into account the possibility of substituting equivalent features.

2.2. Visual merchandising

Visual merchandising is a marketing strategy that promotes the sale of products by displaying them in retail stores. Promoting the sale of a product or service requires combining commodities, experiences, and locations into a stimulating and engaging presentation. Window displays, signs, interior displays, beauty promos, and any other special sales promotions that occur are all covered by visual merchandising (Thakur, 2013). In general, shoppers in supermarkets make quick decisions. In a congested store, product visibility is critical. Imam and Alvi (2017) focused their research on determining the influence of shelf space on consumer decision-making. The scope is to gain a better understanding of customer behavior within the business. The results of the study revealed that alternative product configurations on shelf space have a beneficial influence on in-store buying decisions.

Ali Soomro, Abbas Kaimkhani, and Iqbal (2017) researched to see how aesthetic merchandising affects client attention in a retail business. They examined the impact of influential promotional variables such as window display, store layout, color, and interior illumination on different retail outlets. They concluded that visual merchandising is an important technique that has a significant impact on impulse buying and customer purchasing behavior.

According to Chandon, Hutchinson, Bradlow, and Young (2009), visual merchandising is one of the store marketing methods that causes customers to

make unplanned purchases. Retailers use this method to visually optimize their store by establishing an appealing environment, appropriate shelf arrangement, appealing window display, coherence, and so on.

2.3. Horizontal versus vertical shelf placements

Consumers frequently know what kind of goods they want to buy before coming to the store, but they do not know which variant of the product is preferable at that time. People finally choose the product because it meets their demands at that moment. As a result, a customer may find a good product in one store but decide not to buy it because they want to keep looking for a better one (Cachon, Terwiesch, & Xu 2005). Food products have an expiration date and each store makes its own decisions about the discounts without consulting on the higher level. That is why customers compare prices on assortment at each store, even in the age of the internet. Therefore, the visible product categorization shows the available goods and makes it easier to perform the comparison and evaluation.

With capacity restrictions and a given demand, Corsten, Hopf, Kasper, and Thielen (2018) developed a stylized model for the regionalized assortment planning problem. A common selection is chosen, which is enhanced by regionalized items. While goods from the common selection are available in all stores, those from the local assortments are unique to each location. Akkaş (2019) proposed that shelf space selection could be used as an operational lever to manage perishable inventory expiry. For that purpose, they describe how shelf space affects expiration, then devise a technique for determining the optimum degree of shelf space that takes this into account.

According to Deng, Kahn, Unnava, and Lee (2016), the ease of thinking or processing efficiency is affected by whether objects are shown horizontally or vertically. Data can be analyzed more efficiently in horizontal displays because the horizontal or binocular vision field matches the horizontal or binocular perception field. Horizontal (vs. vertical) eye movement is made easier by the fact that the human sensory span is broader in the horizontal direction (Shi, Wedel, & Pieters, 2013).

Ozcan and Esnaf (2013) investigated the horizontal and vertical shelf placements and obtained the following findings:

- Moving merchandise from the worst horizontal shelf position to the ideal horizontal position leads to an average of 15 percent boost in sales.
- The average difference in sales between the worst and best vertical positions is more than 39%. This analysis demonstrates that vertical location has a 2.5 times greater impact on product sales than horizontal position.

- Two facings of the visible amount at eye level are more noteworthy than five facings at the bottom shelf. Moving a facing of product from the poorest to the best location in a store, based on horizontal and vertical positions, will increase demand for the product by an average of 60 percent (Ozcan & Esnaf, 2013).

Vertical stimuli typically necessitate additional motion. As a result, horizontal scanning should be more fluid than vertical scanning, allowing more possibilities to be evaluated more quickly, increasing perceptual diversity. These changes in processing fluency happen in less than three seconds. Although with sufficient time, this inefficiency can be easily solved (Kahn & Wansink, 2004).

Because of the highlighted importance of vertical and horizontal product categorization on a planogram, we include these factors in our SSAP model.

2.4. Linear and nonlinear programming in SSAP

Most SSAP models are formulated as mixed-integer nonlinear programming.

Hariga, Al-Ahmari, and Mohamed (2007) and Geismar, Dawande, Murthi, and Sriskandarajah (2015) investigated a general approach to allocating shelf space. Because of the demand rate, a nonlinear goal function was implemented in the formulation of the model. Positioning factor demand function has also been incorporated. There are two sorts of elasticity effects to be concerned about: main space and cross space.

In contrast to previous nonlinear programming research, Hansen et al. (2010) proposed a linear programming model obtained by converting the nonlinear profit function to solve the shelf-space allocation problem optimally.

Yang (2001) provided a basic linear model based on the knapsack problem that only took into account product profitability and demonstrated that the problem is NP-hard.

Hübner and Kuhn (2011) suggested an approach that combines assortment and space allocation into a single model. It uses the traditional shelf space model with substitution effects caused by delisted products. By transforming the mixed-integer non-linear problem into a multi-choice knapsack problem with specified demand values, the proposed modeling approach may address problems involving real category measurements.

A piecewise linearization technique can be used to recreate the mixed-integer nonlinear programming model. A linear mixed-integer programming formulation was presented by Gajjar and Adil (2010), Irion, Lu, JAI-Khayyal, and Tsao (2012), and Tsao et al. (2014), with the optimal objective being an upper bound on the old

model. Gajjar and Adil (2010), on the other hand, omitted cross-space elasticity from the mixed-integer nonlinear programming model. Irion et al. (2012) and Tsao, Lu, An, Al-Khayyal, Lu, and Han (2014) used negative cross-space elasticity. When a positive influence is taken into account, the imbalance grows.

The linear models with all of the constraints can be used in the optimization software.

3. Research methodology and problem definition

3.1. Research procedure and methods

The research is provided according to the following steps:

1. Literature gathering on the shelf space allocation and merchandising rules specific to categorisation.
2. Defining the parameters according to which visually attractive categories could be built.
3. Defining the allocation rules.
4. Formulating SSAP, which is nonlinear.
5. Analysing the pros and cons of the SSAP model and reformulating it in a linear form.
6. Experimenting, and comparing the performance of both models.
7. Analysing the results of research and formulating the conclusions.

The research is mainly based on operational research. Simulation, mathematical optimization, and linear and nonlinear programming methods (Duckworth, 2012) are mainly used.

3.2. Problem definition

Parameters and indexes used in the model:

- S – the total number of shelves,
- P – the total number of products,
- K – the total number of categories,
- T – the total number of tags,
- i – shelf index, $i = 1, \dots, S$,
- j – product index, $j = 1, \dots, P$,
- k – category index, $k = 1, \dots, K$,
- t – tag index, $t = 1, \dots, T$,
- r – orientation index, $r \in \{0,1\}$,

$$r = \begin{cases} 0, & \text{for front orientation} \\ 1, & \text{for side orientation} \end{cases}.$$

Shelf parameters:

s_i^l – the length of the shelf i ,

s_i^d – the depth of the shelf i ,

s_{it}^g – binary tag t of the shelf i ,

$$s_{it}^g = \begin{cases} 1, & \text{if shelf } i \text{ is tagged} \\ 0, & \text{otherwise} \end{cases}.$$

Product parameters:

p_j^w – the width of the product j ,

p_j^d – the depth of the product j ,

p_{jr}^w – the width or depth of the product j on orientation r ,

$$p_{jr}^w = \begin{cases} p_{j0}^w, & \text{if } r = 0, \text{ width for front orientation} \\ p_{j1}^w, & \text{if } r = 1, \text{ depth for side orientation} \end{cases},$$

p_j^u – the unit profit of the product j ,

p_j^l – the cluster of the product j ,

$p_j^{o_2}$ – the side orientation binary parameter of the product j ,

$$p_j^{o_2} = \begin{cases} 1, & \text{if side orientation is available for product } j \\ 0, & \text{otherwise} \end{cases},$$

p_j^k – the category of the product j ,

p_{ij}^t – tag t of the product j ,

f_j^{\min} – the minimum number of facings of the product j ,

f_j^{\max} – the maximum number of facings of the product j .

Category parameters:

c_k^m – minimum category size as a percentage of the shelf length,

c_k^t – category size tolerance between shelves in the category as a percentage of the shelf length.

Tag parameters:

b_t^n – the band name of the tag t , $b_t^n = \{H; H^+; V^+\}$,

b_{ij}^t – product to shelf compatibility tag,

$$b'_{ij} = \begin{cases} 1, & \text{if } s'_{ii} = p'_{ij} \wedge b'_t^n = \{H\} \\ 0, & \text{otherwise} \end{cases}, t = 1, \dots, T \text{ – for the horizontal can level shelves,}$$

$$b'_{ij} = \begin{cases} \min(p'_{ij}; 1) \wedge b'_t^n = \{V^+\} \\ 1, & \text{if } p'_{ij} = 1 \wedge s'_{ii} = p'_{ij} \wedge b'_t^n = \{H^+\} \\ 0, & \text{if } p'_{ij} = 1 \wedge s'_{ii} \neq p'_{ij} \wedge b'_t^n = \{H^+\} \\ 1, & \text{if } p'_{ij} = 0 \wedge b'_t^n = \{H^+\} \end{cases}, t = 1, \dots, T \text{ – for the horizontal and}$$

vertical bottle shelves.

Decision variables in nonlinear integer SSAP:

$$x_{ij} = \begin{cases} 1, & \text{if product } j \text{ is put to the shelf } i \\ 0, & \text{otherwise} \end{cases},$$

f_{ij} – the number of facings of the product j on the shelf i ,

$$y_{ij}^{o_1} = \begin{cases} 1, & \text{if product } j \text{ is put to the shelf } i \text{ on front orientation} \\ 0, & \text{otherwise} \end{cases},$$

$$y_{ij}^{o_2} = \begin{cases} 1, & \text{if product } j \text{ is put to the shelf } i \text{ on side orientation} \\ 0, & \text{otherwise} \end{cases}.$$

Decision variables in linear integer SSAP:

$$x_{ijr} = \begin{cases} 1, & \text{if product } j \text{ is placed on shelf } i \text{ on orientation } r \\ 0, & \text{otherwise} \end{cases} \text{ – product placement}$$

binary variable

$$(\text{for all } i = 1, \dots, S, j = 1, \dots, P, r \in \{0, 1\}): \quad x_{ijr} \in \{0, 1\}$$

f_{ijr} – the number of facings of the product j on the shelf i on orientation r ,

$$y_j = \begin{cases} 0, & \text{if product } j \text{ is on front orientation} \\ 1, & \text{if product } j \text{ is on side orientation} \end{cases} \text{ – orientation of the product } j$$

$$(\text{for all } j = 1, \dots, P): \quad y_j \in \{0, 1\}.$$

The investigated SSAP consists of a planogram that is divided vertically into categories. Shelves of a planogram are tagged horizontally. The problem can be formulated as follows: there is a given number of products P that must be placed on S shelves of a planogram. The products are assigned to K categories. The merchandiser allocates each category on a planogram, i.e., they define the category sequences (from left to right). Therefore, there is space initially assigned for each of the K categories, i.e., the minimum category size, allowing it to be visible enough to the customers. The goal is to define the appropriate shelf space for each category that exists on a planogram with regard to the number of facings of each product, maximising retailers' profit.

Products are divided into categories based on their types or classes. Each category is vertical. In addition, the products and shelves are tagged horizontally. Each product could have several tags p_{ij}^t simultaneously. Each shelf could also have several tags s_{ii}^t simultaneously. An example for shelves: (1) a shelf is for a specific product package (can, bottle, pack, shrinkwrap); (2) a shelf is for promo products; (3) a shelf is on eye level; (4) a shelf is on touch level. An example for products: (1) a product is a can, and it must be placed on the shelf for cans; (2) a product is a bottle, and it must be placed on the shelves for bottles on eye-level; (3) a product is a bottle, and it must be placed on the promo shelf for bottles. Collecting products into vertical (categories) and horizontal (bottles, cans, packs) bands results in their better visibility.

There are three possible tags, $b_i^n = \{H; H^+; V^+\}$. The shelves and products may be tagged by T tags. Each shelf or product could be tagged by one or more tags:

- H – the shelf horizontally is dedicated only to specific products (such as cans, bottles, packs, shrinkwraps).
- H^+ – the shelf horizontally is dedicated to specific products of different types (such as promo shelf, eye-level shelf). So cans, as well as bottles, could be placed on promo or eye-level shelves.
- V^+ – the shelf vertically is dedicated to the specific product category. For the vertical product category, several or all shelves could be used to allocate products by brand (such as Cola, Fanta, Sprite).

There is an example case for the merchandiser to allocate the products. For some of them, the tags are specified:

- product 1: brand (V^+) is Cola, package (H) is a bottle, a dedicated shelf is eye-level (H^+);
- product 2: brand (V^+) is Cola, package (H) is can;
- product 3: brand (V^+) is Sprite, package (H) is bottle;
- product 4: brand (V^+) is Sprite, package (H) is can;
- product 5: brand (V^+) is Fanta, package (H) is a bottle, a dedicated shelf is promo-level (H^+);
- product 6: brand (V^+) is Fanta, package (H) is can.

This follows the practice frequently observed in real retail stores. Figure 1 and Figure 2 show the specific nature of the vertical and horizontal bands on a planogram in the investigated problem. In Figure 1, the above-mentioned products in bottles 1, 3, and 5 are marked with a white label. Other bottles without labels (e.g., Light Cola, Light Sprite, Light Fanta) do not have shelf level requirements; there-

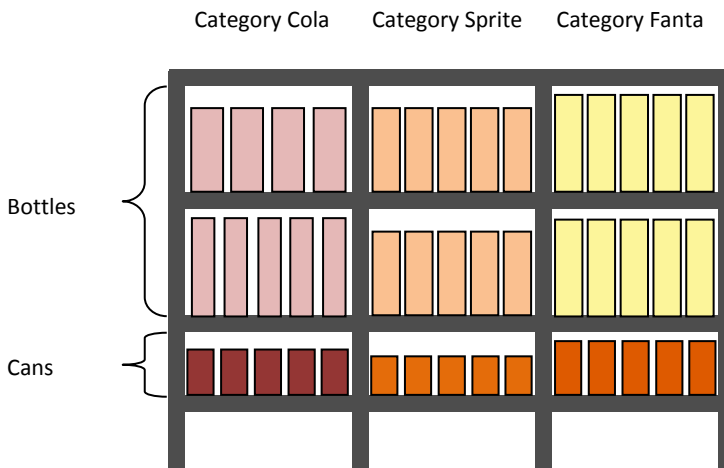
fore, they could be placed on any shelf. They must only be placed on any shelf inside the appropriate vertical category. The lowest shelf is dedicated to bottles. Figure 2 shows the vertical categories. Note that there are different products within one category, e.g., Cola on the middle shelf has five facings and Light Cola on the upper shelf has four facings. Bottles are indicated in a lighter color, and cans are in a darker color. The color of the same category is in a similar tone.

Figure 1. Planogram with vertical and horizontal bands



Source: Authors' own elaboration.

Figure 2. Planogram with vertical and horizontal bands.



Source: Authors' own elaboration.

One product category is allocated to several shelves. However, only one product could be placed on one shelf. The minimum and maximum numbers of facings of each product on the shelf to make it visible enough for the customers are defined by the merchandiser.

There are two kinds of orientation in which the product could be placed on the shelf: front and side ones. By default, front orientation is available for all products. The orientation binary parameter $p_j^{o_2}$ defines whether the product could be placed on side orientation based on the package and brand visibility printed on the package.

The planogram could be more complicated. Based on the cluster parameter p_j^l , some products could be grouped into clusters; therefore, they must be placed on one shelf. This allocation rule ensures the substitution effect between cluster products. Cluster products are not shown in the above-explained figures.

In the current research, we investigated only the front visible facings row. The vertical number of facings and the number of facings in depth were not considered. The shelf depth differs because, in practice, the lower shelves of a planogram are deeper, but the product's depth and the shelf depth were also considered only for the front facings row. If the depth of the shelf is exceeded for the product, and if both front and side orientations of the product are available, this product, in this case, could be rotated on this shelf or placed on a deeper shelf.

To solve the problem, there is a task to decide if the product is placed on the shelf, define the number of facings of each product allocated on each shelf, find if it is placed on the front or side orientation and consider a set of constraints, which we grouped into four categories: shelf constraints, product constraints, orientation constraints, and bands constraints. The goal was to maximize the total profit from allocating products on a planogram.

In this research, we formulated nonlinear (SSAP-NL) and linear (SSAP-L) problems for the given SSAP definition, which differ by decision variables.

In the first nonlinear problem formulation, we had to find:

- x_{ij} – if the product j is placed on the shelf i ,
- f_{ij} – the number of facings of the product j on the shelf i ,
- $y_{ij}^{o_1}$ – if the product j on the shelf i is on front orientation,
- $y_{ij}^{o_2}$ – if the product j on the shelf i is on side orientation.

In the second linear problem formulation, we must find:

- x_{ijr} – if the product j is placed on the shelf i on orientation r ,
- f_{ijr} – the number of facings of the product j on the shelf i on orientation r ,
- y_j – orientation of the product j .

3.3. Nonlinear integer problem formulation

The goal of maximizing the total profit from allocating products on a plagram or the main problem of this research can then be formulated as follows:

$$\max \sum_{j=1}^P \sum_{i=1}^S x_{ij} p_j^u f_{ij} \tag{1}$$

subject to:

1. Shelf constraints

$$\forall(i) [\sum_{j=1}^P f_{ij} (y_{ij}^{o_1} p_j^w + y_{ij}^{o_2} p_j^d) \leq s_i^l] \quad (\text{shelf length}) \tag{2}$$

$$\forall(i, j) [x_{ij} (y_{ij}^{o_1} p_j^d + y_{ij}^{o_2} p_j^w) \leq s_i^d] \quad (\text{shelf depth}) \tag{3}$$

2. Product constraints

$$\forall(j) [\sum_{i=1}^S x_{ij} = 1] \quad (\text{product is placed on one shelf only}) \tag{4}$$

$$\forall(j) [f_j^{\min} \leq \sum_{i=1}^S f_{ij} \leq f_j^{\max}] \quad (\text{minimum and maximum number of facings}) \tag{5}$$

$$\forall(i) \forall(a, b: p_a^l = p_b^l, a, b = 1, \dots, P) [x_{ia} = x_{ib}] \quad (\text{cluster products are placed on the same shelf}) \tag{6}$$

3. Orientation constraints

$$\forall(i, j) [y_{ij}^{o_2} \leq p_j^{o_2}] \quad (\text{side orientation is possible}) \tag{7}$$

$$\forall(i, j) [y_{ij}^{o_1} \cdot y_{ij}^{o_2} = 0] \quad (\text{only one orientation (front or side) is available}) \tag{8}$$

$$\forall(i, j) [y_{ij}^{o_1} + y_{ij}^{o_2} = 1] \quad (\text{only one orientation (front or side) is available}) \tag{9}$$

4. Bands constraints

$$\forall(i, j) [\prod_{t=1}^T b_{ij}^t \geq x_{ij}] \quad (\text{tags compatibility}) \tag{10}$$

$$\forall(i, k) \left[\left(\sum_{\substack{j=1, \\ p_j^k=k}}^P f_{ij} (y_{ij}^{o_1} p_j^w + y_{ij}^{o_2} p_j^d) \geq [s_i^l \cdot c_k^m] \right) \vee \left(\sum_{\substack{j=1, \\ p_j^k=k}}^P f_{ij} = 0 \right) \right], \tag{11}$$

(minimum category size if the category exists on the shelf) (2)

$$\begin{aligned} & \forall(k) \left[\max_{i=1, \dots, S} \left(\sum_{\substack{j=1, \\ p_j^k=k}}^P f_{ij} (y_{ij}^{o_1} p_j^w + y_{ij}^{o_2} p_j^d) \right) - \min_{i=1, \dots, S} \left(\sum_{\substack{j=1, \\ p_j^k=k}}^P f_{ij} (y_{ij}^{o_1} p_j^w + y_{ij}^{o_2} p_j^d) \right) \right] \leq \\ & \leq \left[\max_{i=1, \dots, S} (s_i^l) \cdot c_k^t \right] \end{aligned} \quad ,$$

(category size tolerance) (12)

5. Relationships constraints

$$\forall(i, j) \left[x_{ij} s_i^l \left(\frac{y_{ij}^{o_1}}{p_j^w} + \frac{y_{ij}^{o_2}}{p_j^d} \right) \geq f_{ij} \right] \quad \text{(facings relationships)} \quad (13)$$

$$\forall(i, j) [x_{ij} \leq f_{ij} (y_{ij}^{o_1} + y_{ij}^{o_2})] \quad \text{(facings relationships)} \quad (14)$$

Decision variables:

$$\forall(i, j) [x_{ij} \in \{0, 1\}] \quad \text{(the product is on the shelf)} \quad (15)$$

$$\forall(i, j) [f_{ij} = \{f_j^{\min} \dots f_j^{\max}\}] \quad \text{(the number of facings)} \quad (16)$$

$$\forall(i, j) [y_{ij}^{o_1} \in \{0, 1\}] \quad \text{(front orientation)} \quad (17)$$

$$\forall(i, j) [y_{ij}^{o_2} \in \{0, 1\}] \quad \text{(side orientation)} \quad (18)$$

3.4. Linear integer problem formulation

The problem can be formulated as follows:

$$\max \sum_{j=1}^P \sum_{i=1}^S \sum_{r=0}^1 p_j^u f_{ijr} \quad (19)$$

subject to:

1. Shelf constraints

$$\forall(i) \left[\sum_{j=1}^P \sum_{r=0}^1 p_{jr}^w f_{ijr} \leq s_i^w \right] \quad \text{(shelf length)} \quad (20)$$

$$\forall(i, j, p_{j1}^w > s_i^d) [f_{ij0} = 0] \quad \text{(shelf depth for front orientation)} \quad (21)$$

$$\forall(i, j, p_{j0}^w > s_i^d) [f_{ij1} = 0] \quad \text{(shelf depth for side orientation)} \quad (22)$$

2. Product constraints

$$\forall(j) \left[\sum_{i=1}^S \sum_{r=0}^1 x_{ijr} = 1 \right] \quad \text{(product is placed on one shelf only)} \quad (23)$$

$$\forall(i, j, r)[f_{ijr} \leq x_{ijr} f_j^{\max}] \quad \text{(product is placed on the shelf)} \quad (24)$$

$$\forall(j)[f_j^{\min} \leq \sum_{i=1}^S \sum_{r=0}^1 f_{ijr} \leq f_j^{\max}] \quad \text{(minimum and maximum number of facings)} \quad (25)$$

$$\forall(i)\forall(a, b: p_a^l = p_b^l, a, b = 1, \dots, P)[\sum_{r=0}^1 x_{iar} = \sum_{r=0}^1 x_{ibr}] \quad \text{(cluster products are placed on the same shelf)} \quad (26)$$

3. Orientation constraints

$$\forall(i, j)[y_j \leq p_j^{o_2}] \quad \text{(side orientation is possible)} \quad (27)$$

$$\forall(i, j)[\sum_{r=0}^1 x_{ijr} \leq 1] \quad \text{(only one orientation (front or side) is available)} \quad (28)$$

4. Bands constraints

$$\forall(i, j)[\prod_{i=1}^T b_{ij}^t \geq \sum_{r=0}^1 x_{ijr}] \quad \text{(tags compatibility)} \quad (29)$$

$$\forall(i, k)[(\sum_{j=1}^P \sum_{r=0}^1 p_{jr}^w f_{ijr} \geq [s_i^l \cdot c_k^m]) \vee (\sum_{j=1}^P \sum_{r=0}^1 f_{ijr} = 0)], \quad \text{(minimum category size if the category exists on the shelf)} \quad (30)$$

$$\forall(k)[\max_{i=1, \dots, S} (\sum_{j=1}^P \sum_{r=0}^1 p_{jr}^w f_{ijr}) - \min_{i=1, \dots, S} (\sum_{j=1}^P \sum_{r=0}^1 p_{jr}^w f_{ijr}) \leq [\max_{i=1, \dots, S} (s_i^l) \cdot c_k^t]], \quad \text{(category size tolerance)} \quad (31)$$

5. Relationships constraints

$$\forall(i, j, r)[f_{ijr} \geq x_{ijr}] \quad \text{(facings relationships)} \quad (32)$$

$$\forall(i, j)[f_{ij0} \leq (1 - y_j) f_j^{\max}] \quad \text{(facings and orientation relationships)} \quad (33)$$

$$\forall(i, j)[f_{ij1} \leq y_j f_j^{\max}] \quad \text{(facings and orientation relationships)} \quad (34)$$

6. Decision variables

$$\forall(i, j, r)[x_{ijr} \in \{0, 1\}] \quad \text{(the product is on the shelf)} \quad (35)$$

$$\forall(i, j, r)[f_{ijr} = \{f_j^{\min} \dots f_j^{\max}\}] \quad \text{(the number of facings)} \quad (36)$$

$$\forall(j)[y_j \in \{0, 1\}] \quad \text{(orientation)} \quad (37)$$

4. Research findings

The computational experiments evaluate the profit found by the CPLEX solver for non-linear and linear formulations of the same problem. The difference was only in a formal mathematical representation of the problems. The data were simulated based on real-life store cases.

There were four shelves in a planogram. There were two package types: cans and bottles. The shelf levels were as follows:

- cans only (H),
- bottles on promo-level (H^+),
- bottles on eye-level (H^+),
- bottles of any product (V^+).

There were planograms of five shelf widths of 250, 375, ..., 750 cm. Nine product sets that contained 15, 20, ..., and 50 products had to be placed in a planogram. For the sets of 10, 15, and 20 products, there were two vertical categories. For the sets of 25 and 30 products, there were three vertical categories. For the sets of 35 and 40 products, there were four vertical categories. For the sets of 45 and 50 products, there were five vertical categories.

An optimal solution for SSAP-L and feasible (or in some cases optimal) solution was found using commercial solver IBM ILOG CPLEX Optimization Studio Version: 12.7.1.0.

Table 1 compares the quality of solution of SSAP-NL and SSAP-L formulations found by the CPLEX solver. At first, we found optimal solutions for the SSAP-L in all test cases. Observe that the average computation time is 20 seconds with minimal and maximal values of 1 second and 87 seconds, respectively. Next, we found the solution of the SSAP-NL within 5 minutes' execution time limit. The fourth column shows the solution quality of SSAP-NL in 5 minutes. This column presents the profit ratio of the SSAP-NL to SSAP-L. Observe that in 19 out of 45 cases, the feasible solution was optimal. However, in 25 cases, the profit ratio decreased up to 97.53%. The average profit ratio was 99.58%. Finally, we increased the time limit to 10 minutes and tried to find the solution to the 26 cases where the solution was not optimal. This gave us 16 cases in which the solution was improved. Nevertheless, in nine cases, the solution was found the same as in the 5-minute time limit. The increase of time to 10 minutes gave three optimal solutions which have not been found in the 5-minute time limit. The average profit ratio was 99.43%. This is lower than in the case of the 5-minute limit because we did not take all test cases into account,

only those in which there was no optimal solution found in the 5-minute time limit. The lowest profit ratio now is slightly higher and equals 98.72%. This proves the importance of time limit setting while solving non-linear problems.

Table 1. Comparison of the SSAP-NL and SSAP-L solution

Number of products	Shelf width [cm]	Time for SSAP-L [s]	SSAP-NL/SSAP-L profit ratio in 5 minutes	SSAP-NL/SSAP-L profit ratio in 10 minutes*
<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
10	250	1	100.00%	
	375	5	100.00%	
	500	5	100.00%	
	625	3	100.00%	
	750	4	100.00%	
15	250	6	100.00%	
	375	5	100.00%	
	500	7	100.00%	
	625	3	100.00%	
	750	4	100.00%	
20	250	11	100.00%	
	375	8	100.00%	
	500	6	100.00%	
	625	10	100.00%	
	750	9	99.93%	100.00%
25	250	11	100.00%	
	375	12	100.00%	
	500	11	99.97%	99.97%
	625	12	99.74%	100.00%
	750	74	No solution	No solution
30	250	16	100.00%	
	375	13	99.49%	99.49%
	500	13	99.69%	99.78%
	625	11	99.53%	99.67%
	750	13	99.74%	99.79%
35	250	23	100.00%	
	375	25	99.20%	99.40%
	500	19	98.91%	98.91%
	625	17	99.72%	99.72%
	750	20	99.53%	99.60%
40	250	18	99.87%	100.00%
	375	19	99.71%	99.93%
	500	19	99.25%	99.25%
	625	18	99.28%	99.77%
	750	16	99.53%	99.60%
45	250	19	98.88%	98.88%
	375	25	97.53%	98.72%
	500	53	99.06%	99.06%
	625	41	98.43%	98.86%
	750	46	98.46%	99.20%

Table 1 cont.

<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
50	250	20	100.00%	
	375	46	98.42%	98.78%
	500	87	99.39%	99.44%
	625	39	98.91%	98.91%
	750	36	99.14%	99.14%
Minimum		1	97.53%	98.72%
Average		20	99.58%	99.43%
Maximum		87	100.00%	100.00%

* We increased time to 10 minutes and repeated the experiment only in the instances with the non-optimal solution within a 5-minute time limit. Therefore, some rows are missing data.

Source: Authors' own elaboration.

Interestingly, for the case with 25 products on 750 cm, the solution of the SSAP-NL was neither found in 5 nor in 10 minutes. However, the optimal solution of the SSAP-L was found only in 74 seconds. This proves the importance of SSAP formulation in a linear form.

5. Discussion

Products are placed on the shelves according to specific rules (Cachon et al., 2005; Ebster & Garaus 2015; Hansen et al., 2010; Mowrey, Parikh, & Gue, 2019). Merchandisers use existing standards to allocate them, which differ at each retail point of sale. The correct allocation of specific items helps to influence the buyer. Thus, sales increase and, as a result, so does the company's income.

Categorizing product items is very important, and such categorisation can allow the overall assortment to be examined more easily by customers. Horizontal and vertical allocations are standard allocation methods. Products are placed horizontally or vertically in a planogram. Merchandisers or sellers themselves are responsible for the correct product display. Heads of departments and managers regularly check the compliance of product placement on the trading floor with the target planogram.

In this research, we investigated the planogram with vertical and horizontal categorization, which build visible bands on a planogram. An example of such an approach could be the following: make a sales rating for products of one category (e.g., grocery, dairy products), break it down into groups: milk, yogurt, kefir, cottage cheese, salt, sugar, flour, and others. Group them by package types (e.g., bottles, cartons). Form the visible vertical and horizontal bands in a planogram.

The key characteristic of our model is the vertical categorization of products by a specific characteristic (e.g., type, brand, colour) and simultaneously horizontal categorization by another specific characteristic (e.g., package type). What's more, in our model, we applied the four sets of constraints (shelf constraints, product constraints, orientation constraints, and band constraints).

The linear constraints, including the decision variables, are implemented in the optimization software. The advantages of the proposed linear-formulation model are the flexibility of the evaluations of the problem and the ease of constraints implementation using a commercial solver. CPLEX solver obtained optimal results for large-size problems.

6. Conclusions

The positioning of product categories on store shelves is determined by pattern choices. The horizontal and vertical locations of product items in this retail area are chosen based on criteria such as product category, brand, or package type.

In this research, we enhance the basic planogram profit maximization model with the possibility to simultaneously categorize products vertically and horizontally. The vertical categorization is based on one characteristic (e.g., type, brand, colour). The horizontal categorization is based on another characteristic (e.g., package type).

We compared the quality of the solutions obtained while solving the non-linear and linear models. Optimal solutions were found in all instances of the linear model. For the non-linear model, feasible or optimal solutions were obtained. The quality of the non-linear solution within a 10-minute time limit was quite good and no less than 98.72%. The average quality of the non-linear model in a 10-minute limit was 99.43%.

In one instance, the solution to the non-linear model has not been found. All instances of the linear model were solved in a couple of seconds, up to approximately 1.5 minutes, but mostly less than a minute. This shows the solution time and quality advantages of a linear model over a non-linear one.

The results of this research show that CPLEX finds an extremely good solution for non-linear problems. So, if the problem could not be modeled in a linear form, the solution provided by a commercial solver is quite enough. But if there is an opportunity to develop a model in a linear form (this research presented a method of changing the dimensions of decision variables), it is advised to do so because, in this case, an optimal solution could be obtained.

The experiment proves that the linear problem formulation allows for finding the solution faster. For large instances, the solution received is of better quality. For small instances, both linear and non-linear problem formulations found optimal solutions. The results for linear and non-linear problems are similar to existing research findings (for instance, Gajjar & Adil, 2010; Hariga et al., 2007). The main differences between our research and existing findings rely on the integration of horizontal and vertical rules and including new constraints. The main implication of the research results for science is related to the possibility of determining an optimal solution to the initially formulated non-linear shelf space allocation problem including simultaneous categorization of products on horizontal and vertical rules. The main implication for practice is considering the practical constraints based on customers' requirements. The results of research are universal and can be implemented in any retail information system.

The main limitations of this research are not including the restrictions on storage conditions of specific products (e.g., fresh, frozen, or refrigerated products) and holding time for perishable products in the models.

The future research directions could be the following:

- Investigate customers' spontaneous reactions to assortment based on stimuli explained in this research, such as horizontal and vertical bands in a planogram.
- Investigate the relation between the band color in a planogram, occupied shelf space by the products of the definite band color category, and the level of customer's attraction to such a planogram.
- Develop measurement techniques such as eye-tracking measurement, which evaluate the customers' reactions to visible product categorization.
- Investigate the effects of top-down visualization stimuli in the purchasing process.
- Include a turnover ratio, which is a very important factor in retail.
- The influence of virtual services on customers' behavior (for instance, the influence of COVID on changing the work and lifestyle of customers).

Learning how all of these aspects interact will allow the store to better merchandise its assortment offered to different types of customers. Last but not least, more problem scenarios should be solved in the future to highlight the effectiveness of the proposed model formulation and solution. Moreover, constraints for storage conditions, holding costs, and time for perishable items should be added. Not only does this extension allow the application of the proposed models to planograms with packed products stored at room temperature, but also to other planogram fixture types (e.g., refrigerator, fresh fruits, and vegetable bins) and to display products either in a specific wrap or without a wrap at all.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The project is financed by the Ministry of Science and Higher Education in Poland under the program “Regional Initiative of Excellence” 2019-2022 project number 015/RID/2018/19, total funding amount 10,721,040.00 PLN.

References

- Imam, A. A. A., & Alvi, A. K. (2015). Effect of shelf space optimisation on consumer decision making. *Science International*, 27(1), 439-444. Retrieved from <https://www.thefreelibrary.com/EFFECT+OF+SHELF+SPACE+OPTIMIZATION+ON+CONSUMER+DECISION+MAKING-a0405930870>
- Akkaş, A. (2019). Shelf space selection to control product expiration. *Production and Operations Management*, 28(9), 2184-2201. <https://doi.org/10.1111/poms.13034>
- Ali Soomro, D. Y., Abbas Kaimkhani, S., & Iqbal, J. (2017). Effect of visual merchandising elements of retail store on consumer attention. *Journal of Business Strategies*, 11(1), 21-40. Retrieved from <https://www.proquest.com/docview/1925865276/fulltextPDF/CEF9BD32F1D5481BPQ/1?accountid=45580>
- Bianchi-Aguiar, T., Silva, E., Guimarães, L., Carravilla, M. A., & Oliveira, J. F. (2018). Allocating products on shelves under merchandising rules: Multi-level product families with display directions. *Omega*, 76, 47-62. <https://doi.org/10.1016/j.omega.2017.04.002>
- Borin, N., Farris, P. W., & Freeland, J. R. (1994). A model for determining retail product category assortment and shelf space allocation. *Decision Sciences*, 25(3), 359-384. Retrieved from <https://onlinelibrary.wiley.com/doi/10.1111/j.1540-5915.1994.tb00809.x>
- Cachon, G. P., Terwiesch, C., & Xu, Y. (2005). Retail assortment planning in the presence of consumer search. *Manufacturing and Service Operations Management*, 7(4), 330-346. <https://doi.org/10.1287/msom.1050.0088>
- Chandon, P., Hutchinson, J. W., Bradlow, E. T., & Young, S. H. (2009). Does in-store marketing work? Effects of the number and position of shelf facings on brand attention and evaluation at the point of purchase. *Journal of Marketing*, 73(6), 1-17. <https://doi.org/10.1509/jmkg.73.6.1>
- Corsten, H., Hopf, M., Kasper, B., & Thielen, C. (2018). Assortment planning for multiple chain stores. *OR Spectrum*, 40(4), 875-912. <https://doi.org/10.1007/s00291-017-0496-9>
- Corstjens, M., & Doyle, P. (1981). A model for optimising retail space allocations. *Management Science*, 27(7), 822-833. <https://doi.org/10.1287/mnsc.27.7.822>

- Deng, X., Kahn, B. E., Unnava, H. R., & Lee, H. (2016). "Wide" variety: Effects of horizontal versus vertical display on assortment processing, perceived variety, and choice. *Journal of Marketing Research*, 53(5), 682-698. <https://doi.org/10.1509/jmr.13.0151>
- Dhar, S. K., Hoch, S. J., & Kumar, N. (2001). Effective category management depends on the role of the category. *Journal of Retailing*, 77(2), 165-184. [https://doi.org/10.1016/S0022-4359\(01\)00045-8](https://doi.org/10.1016/S0022-4359(01)00045-8)
- Duckworth, W. E. (2012). *A guide to operational research*. Berlin/Heidelberg: Springer Science & Business Media.
- Düsterhöft, T., Hübner, A., & Schaal, K. (2020). A practical approach to the shelf-space allocation and replenishment problem with heterogeneously sized shelves. *European Journal of Operational Research*, 282(1), 252-266. <https://doi.org/10.1016/j.ejor.2019.09.012>
- Ebster, C., & Garaus, M. (2015). Chapter 4. Visual merchandising: Capturing customer attention. In C. Ebster & M. Garaus, *Store design and visual merchandising: Creating store space that encourages buying* (2nd ed.; pp. 83-114). New York, NY: Business Expert Press. <https://doi.org/10.4128/9781606490952>
- Gajjar, H. K., & Adil, G. K. (2010). A piecewise linearisation for retail shelf space allocation problem and a local search heuristic. *Annals of Operations Research*, 179(1), 149-167. <https://doi.org/10.1007/s10479-008-0455-6>
- Geismar, H. N., Dawande, M., Murthi, B. P. S., & Sriskandarajah, C. (2015). Maximizing revenue through two-dimensional shelf-space allocation. *Production and Operations Management*, 24(7), 1148-1163. <https://doi.org/10.1111/poms.12316>
- Ghazavi, E., & Lotfi, M. M. (2016). Formulation of customers' shopping path in shelf space planning: A simulation-optimization approach. *Expert Systems with Applications*, 55, 243-254. <https://doi.org/10.1016/j.eswa.2016.01.043>
- Hansen, J. M., Raut, S., & Swami, S. (2010). Retail shelf allocation: A comparative analysis of heuristic and meta-heuristic approaches. *Journal of Retailing*, 86(1), 94-105. <https://doi.org/10.1016/j.jretai.2010.01.004>
- Hariga, M. A., Al-Ahmari, A., & Mohamed, A.-R. A. (2007). A joint optimisation model for inventory replenishment, product assortment, shelf space and display area allocation decisions. *European Journal of Operational Research*, 181(1), 239-251. <https://doi.org/10.1016/j.ejor.2006.06.025>
- Hübner, A. H., & Kuhn, H. (2011a). Retail shelf space management model with space-elastic demand and consumer-driven substitution effects. *SSRN Electronic Journal*. <https://doi.org/10.2139/ssrn.1534665>
- Hwang, H., Choi, B., & Lee, G. (2009). A genetic algorithm approach to an integrated problem of shelf space design and item allocation. *Computers and Industrial Engineering*, 56(3), 809-820. <https://doi.org/10.1016/j.cie.2008.09.012>
- Irion, J., Lu, J. C., Al-Khayyal, F., & Tsao, Y.-C. (2012). A piecewise linearisation framework for retail shelf space management models. *European Journal of Operational Research*, 222(1), 122-136. <https://doi.org/10.1016/j.ejor.2012.04.021>

-
- Kahn, B. E. (2017). Using visual design to improve customer perceptions of online assortments. *Journal of Retailing*, 93(1), 29-42. <https://doi.org/10.1016/j.jretai.2016.11.004>
- Kahn, B., & Wansink, B. (2004). The influence of assortment structure on perceived variety and consumption qualities. *Journal of Consumer Research*, 30(4), 519-533. <https://doi.org/10.1086/380286>
- Landa-Silva, D., Marikar, F., & Le, K. (2009). Heuristic approach for automated shelf space allocation. In *ISAC'08: Proceedings of the ACM Symposium on Applied Computing* (pp. 922-928). New York, NY: Association for Computing Machinery. <https://doi.org/10.1145/1529282.1529482>
- Lim, A., Rodrigues, B., & Zhang, X. (2004). Metaheuristics with local search techniques for retail shelf-space optimisation. *Management Science*, 50(1), 117-131. <https://doi.org/10.1287/mnsc.1030.0165>
- Mowrey, C. H., Parikh, P. J., & Gue, K. R. (2019). The impact of rack layout on visual experience in a retail store. *INFOR*, 57(1), 75-98. <https://doi.org/10.1080/03155986.2017.1402475>
- Ozcan, T., & Esnaf, S. (2013). A discrete constrained optimization using genetic algorithms for a bookstore layout. *International Journal of Computational Intelligence Systems*, 6(2), 261-278. <https://doi.org/10.1080/18756891.2013.768447>
- Shi, S. W., Wedel, M., & Pieters, F. G. M. (Rik) (2013). Information acquisition during online decision making: A model-based exploration using eye-tracking data. *Management Science*, 59(5), 1009-1026. <https://doi.org/10.1287/mnsc.1120.1625>
- Thakur, A. (Ed.) (2013). *Visual merchandising*. Phagwara: Lovely Professional University. Retrieved from http://ebooks.lpude.in/management/mba/term_4/dmgt552_visual_merchandising.pdf
- Tsao, Y. C., Lu, J. C., An, N., Al-Khayyal, F., Lu, R. W., & Han, G. (2014). Retailer shelf-space management with trade allowance: A Stackelberg game between retailer and manufacturers. *International Journal of Production Economics*, 148, 133-144. <https://doi.org/10.1016/j.ijpe.2013.09.018>
- Yang, M. H. (2001). Efficient algorithm to allocate shelf space. *European Journal of Operational Research*, 131(1), 107-118. [https://doi.org/10.1016/S0377-2217\(99\)00448-8](https://doi.org/10.1016/S0377-2217(99)00448-8)
- Yang, M.-H., & Chen, W.-C. (1999). Study on shelf space allocation and management. *International Journal of Production Economics*, 60, 309-317. [https://doi.org/10.1016/s0925-5273\(98\)00134-0](https://doi.org/10.1016/s0925-5273(98)00134-0)