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Ranking of optimal stock portfolios determined on the basis of expected utility maximization criterion

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Abstract

Aim/purpose – The aim of the paper is to rank the optimal portfolios of shares of companies listed on the Warsaw Stock Exchange, taking into account the investor's propensity to risk.

Design/methodology/approach – Investment portfolios consisting of varied number of companies selected from WIG 20 index were built. Next, the weights of equity holdings of these companies in the entire portfolio were determined, maximizing portfolio's expected (square) utility function, and then the obtained structures were compared between investors with various levels of risk propensity. Using Hellwig's taxonomic development measure, a ranking of optimum stock portfolios depending on the investor's risk propensity was prepared. The research analyzed quotations from 248 trading sessions.

Findings – The findings indicated that whilst there are differences in the weight structures of equity holdings in the entire portfolio between the investor characterized by aversion to risk at the level of $\gamma = 10$ and the investor characterized by aversion to risk at the level of $\gamma = 100$, the rankings of the constructed optimum portfolios demonstrate strong similarity. The study validated, in conformity with the literature, that with the increase in the number of equity holdings in the portfolio, the portfolio risk initially decreases and then becomes stable at a certain level.

Research implications/limitations – The study used data from the past as for which there is no guarantee that they will be adequate for the future. There is sensitivity to the selection of the period from which the historic data come. When changing the period of the analyzed historic data by a small time unit it may prove that the portfolio composition will become totally different.

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Originality/value/contribution – The paper compares the composition of optimum stock portfolios depending on the investor's propensity to risk. Their ranking was created using the taxonomic method for this purpose. Taking advantage of this method also additional variables can be taken into account, which describe and differentiate the portfolio and they can be assigned relevant significance depending on the investor's preferences.

Keywords: optimal portfolio, expected rate of return on the portfolio, portfolio standard deviation, expected utility theory, multidimensional comparative analysis.
JEL Classification: G10, G11.

1. Introduction

The essence of every investment is multiplication of the invested capital. When investing on the Stock Exchange, the investor should make investment decisions based on reasonable grounds. In this respect, the methods of stock market analysis become of great importance, which in addition to stock price analysis, fundamental analysis and behavioral analysis, include portfolio analysis.

In 1952 an article entitled *Portfolio selection* by Markowitz (1952), gave rise to the development of the entire portfolio theory. Author believed that for the investor striving to achieve the highest possible yield, investing only in a single asset with a high rate of return is risky and irrational. Assets with a high rate of return are characterized by large fluctuations in their prices, which is associated with a high risk of loss. In order to limit this (specific) risk, the investor should diversify the investment portfolio. Portfolio content optimization is thereby a two-criteria task in which the return on the portfolio is maximized while minimizing the investment risk. The investor thus has two conflicting goals that should be balanced.

The problem of choosing the optimal stock portfolio can be determined based on the criterion of maximizing the expected (quadratic) utility function (e.g., Bodnar, Okhrin, Vitlinskyy, & Zabolotskyy, 2018; Bodnar & Schmid, 2008, 2009, 2011; Kourtis, Dotsis, & Markellos, 2012; Okhrin & Schmid, 2006; Septiano, Syafriand, & Rosha, 2019). Okhrin & Schmid (2006) obtained the first two moments of the estimated weights of the portfolio, whereas Bodnar & Schmidt (2008, 2009, 2011) derived the sample distributions of its estimated expected return and variance. According to the expected utility hypothesis, the investor will choose such a composition of stock weights in the portfolio that the expected value of the utility function is the maximum (Chopra & Ziemba, 2011). As Merton (1980) points out, it is important to consider the effect of changes in the investor's risk level. The risk aversion ratio is a comparative measure of the risk tolerance level. According to Ross (1981), it is used to compare the investor's behavior in situations of taking risky choices. Changing the value of the risk aversion ratio allows one to analyze the weights of any portfolio.

The quadratic utility function is commonly applied in portfolio theory because of its nice mathematical properties. First, an analytic solution is easy to obtain for the quadratic utility function. Second, Tobin (1958) showed that the Bernoulli principle is satisfied for the mean-variance solution only if one of the following two conditions is valid: the asset returns are normally distributed or the utility function is quadratic. Moreover, the quadratic utility presents a good approximation of other utility functions (e.g., Brandt & Santa-Clara, 2006; Kroll, Levy, & Markowitz, 1984).

Regarding the literature query made, it was noted that the optimal equity portfolios determined on the basis of the criterion of maximizing the expected (quadratic) utility function, and taking into account different levels of the risk aversion coefficient, were built with the use of companies listed on the American (Duan, 2007) and the Indonesian stock exchange (Farkhati, Hoyyi, & Wilandari, 2014; Septiano et al., 2019). These studies analyzed, depending on the risk aversion ratio, the weight of shares in optimal portfolios composed of five and twelve companies. However, there are no such studies with the use of companies listed on the Warsaw Stock Exchange. Moreover, there is no ranking of the optimal stock portfolios (composed of a different number of components) determined on the basis of the criterion of maximizing the expected (quadratic) utility function, either. Therefore, this study tried to fill the existing research gap.

The aim of the paper is to rank the optimal portfolios of shares of companies listed on the Warsaw Stock Exchange, taking into account the investor's propensity to risk because under the same conditions different investors make different decisions, more or less risky.

The following research questions were formulated as a starting point for the considerations:

1. What is the structure of stock weights in portfolios (composed of different numbers of components) that maximize the expected (quadratic) utility function of the investment, with the adopted risk-aversion coefficient at the level of $\gamma = 10$?
2. What is the structure of stock weights in portfolios (composed of different numbers of components) that maximize the expected (quadratic) utility function of the investment, with the adopted risk-aversion coefficient at the level of $\gamma = 100$?

3. Whether and to what extent are the structures of share weights similar in the constructed optimal portfolios (composed of different numbers of components) between investors with different risk aversion at the level of $\gamma = 10$ and $\gamma = 100$?
4. What is the expected rate of return on each portfolio and what is their risk?
5. Are the rankings of optimal stock portfolios (taking into account both the expected rate of return on the portfolio and its risk) for investors with risk aversion at the level of $\gamma = 10$ and $\gamma = 100$ similar to each other? And how much?

The study consists of six parts, one theoretical and five empirical. The first part presents a review of literature, including the basic parameters of the stock portfolio and the task of maximizing the expected (quadratic) utility function. The second part presents the expected rate of return and the standard deviation of rates of return of shares of companies listed on the Warsaw Stock Exchange and selected for the study. In the third part, portfolios consisting of various numbers of components were constructed. The portfolios were constructed on the basis of the criterion of the lowest average value of the correlation coefficient between pairs of daily simple rates of return (group one) and on the basis of the criterion of the highest average value of expected rates of return on shares (group two). The fourth part presents weights of the shares of optimal portfolios, determined on the basis of the criterion of maximizing the expected (quadratic) utility function. Then, the weight structures in optimal equity portfolios were compared between investors with risk aversion at the level of $\gamma = 10$ and $\gamma = 100$. The fifth part presents the values of the expected rate of return and the standard deviation of each portfolio depending on the level of the investor's risk aversion. The last sixth section compares the rankings of optimal equity portfolios between investors with different levels of risk aversion. A synthetic measure was used to prepare the rankings – a taxonomic measure of development by Hellwig (1968).

2. Literature review

2.1. The state of research on portfolio optimization

The concepts of portfolio optimization and diversification have been instrumental in the development and understanding of financial markets and financial decision making. The pioneering work of Markowitz in which he presented a revolutionary approach to portfolio theory called the mean-variance model was a breakthrough. Over time, this model has been expanded to meet the challenges of applying portfolio optimization in practice. These extensions include, e.g., the inclusion of transaction costs (such as market impact costs) and tax effects (Almgren, Thum, Hauptmann, & Li, 2005; Hasbrouck, 1991; Lillo, Farmer, & Mantegna, 2003), the addition of various types of constrains that take specific investment guidelines and institutional features into account (Clarke, De Silva, & Thorley, 2002; Scherer & Xu, 2007), modeling and quantification of the impact of estimation errors in risk and return forecasts on the portfolios via Bayesian techniques, stochastic optimization, or robust optimization approaches (Cremers, Kritzman, & Page, 2005; Tütüncü & Koenig, 2004), and practical multi-period optimization (Boyd, Mueller, O'Donoghue, & Wang, 2013). The problem of optimizing the securities portfolio is thoroughly presented in a monographic article (Kolm, Tutuncu, & Fabozzi, 2014).

Over the past decade, researchers have analyzed current trends and future lines of research into portfolio optimization. In this context, Azmi & Tamiz (2010) reviewed lexicographic, weighted, minmax and fuzzy goal programing models and discussed the issues concerning multi-period returns, extended factors and measurement of risk. Metaxiotis & Liagkouras (2012) and Ponsich, Jaimes, & Coello Coello (2013) analyzed the current state of research in portfolio optimization with a focus on Multi Objective Evolutionary Algorithms in which the lack of many real-life constraints as well as ineffectiveness of Pareto ranking schemes in the presence of many objectives are indicated. Mansini, Ogryczak, & Speranza (2014) focused on linear programing solvable models in the portfolio optimization classifying the models according to decision variables used in the integration of real features. Kolm, Tutuncu, & Fabozzi (2014) discussed practical advances in optimization portfolio and pointed out new research directions such as diversification methods and multi-period optimization. Aouni, Colapinto, & La Torre (2014) reviewed the lexicographic, weighted, polynomial, stochastic and fuzzy goal programing models and pointed out the lack in developing computerized decision support systems to accomplish a helpful tool to facilitate the decision-making process in portfolio optimization. Doering, Juan, Kizys, Fito, & Calvet (2016) focused on recent contributions of metaheuristics in the sense of an introduction to this topic supported with a numerical example. Masmoudi & Abdelaziz (2018) focused on deterministic and stochastic multi-objective programing models comparing the different assumptions and proposed solutions in portfolio optimization. Zhang, Li, & Guo (2018) reviewed various extensions of Markowitz's mean-variance model, such as dynamic, robust, fuzzy portfolio optimization with practical factors and pointed out that combined forecasting theory with portfolio. Aouni, Doumpos, Pérez-Gladish, & Steuer (2018) reviewed multiple criteria decision aid methods for portfolio selection with a focus on exact solution methods on the construction and optimization of portfolios as well as on the analysis and the evaluation of specific securities.

2.2. Portfolio weights maximizing the expected (square) utility of the portfolio

2.2.1. Basic parameters of the share portfolio

In the Markowitz model, the expected rate of return is used as the measure of income, and the variance – or, which is equivalent, the standard deviation of the rates of return, is used as the measure of risk. Using historical data, the expected value of the rate of return for the *i*-th share is determined with the formula (Pera, Buła, & Mitrenga, 2014):

$$E\mathrm{r_i} = \frac{1}{\mathrm{n}}\sum_{\mathrm{k=1}}^{\mathrm{n}} \mathrm{r_{ik}} \tag{1}$$

here:
r_{ik} – rate of return on the *i*-th share achieved in the *k*-th period,
n – number of periods from which the data come.

Accordingly, the variance (standard deviation) of the rate of return for the i-th action can be calculated using the following formulas (Pera et al., 2014):

$$\sigma_i^2 = \frac{1}{n}\sum_{k=1}^{n}(r_{ik} - E_{ri})^2 \tag{2}$$

$$\sigma_i = \sqrt{\sigma_i^2} \tag{3}$$

where:
σ_i^2 – variance of the rate of return for the i-th share,
σ_i – standard deviation of the rate of return for the i-th share.

Therefore, $\mu = [\mu_1, \mu_2, \dots, \mu_n]'$ denotes a column vector of expected rates of return with $\mu_i = Er_i$ elements, while $w = [\omega_1, \omega_2, \dots, \omega_n]'$ denotes a column vector of weights defining the wallet structure. The covariance and variance of individual rates of return can be written as a variance-covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \cdots & cov_{1n} \\ \cdots & \ddots & \ldots \\ cov_{nj} & \ldots & \sigma_{nn}^2 \end{bmatrix} \tag{4}$$

where cov_{ij} denotes the covariance between r_i and r_j. The following relationship was used when calculating the covariance (Pera et al., 2014):

$$cov(r_i, r_j) = \frac{1}{n}\sum_{k=1}^{n}(r_{ik} - E_{ri})(r_{jk} - E_{rj}) \tag{5}$$

Therefore, the expected value of rate of return on share portfolio (μ_P) as the weighted average of the rates of return of its individual components, where the weight is the percentage of a given component in the total portfolio capitalization, is defined as (Pera et al., 2014):

$$\mu_p = \sum_{i=1}^{n} \omega_i \, Er_i \tag{6}$$

or in matrix form:

$$\mu_p = w'\mu \tag{7}$$

In turn, the variance of a share portfolio (σ_p^2), which is not the weighted average of deviations of its individual components, is defined as (Pera et al., 2014):

$$\sigma_p^2 = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \omega_i \, \omega_j \sigma_i \sigma_j cov(r_i, r_j) = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i \, \omega_j cov(r_i, r_j) \tag{8}$$

or in matrix form:

$$\sigma_p^2 = w'\Sigma w \tag{9}$$

2.2.2. Weights maximizing the expected utility of portfolio

The problem of maximizing the expected (quadratic) portfolio utility function ($\overline{U}$) can be written as follows (Okhrin & Schmid, 2006; Kourtis et al., 2012)[1]:

$$\overline{U} = \mu_P - \frac{\gamma}{2}\sigma_p^2 \underset{\vec{\omega}}{\rightarrow} \max \tag{10}$$

[1] In Kim, Kim, & Fabozzi (2014) the function written as $\overline{U} = \frac{1}{2}\sigma_P^2 - \lambda\mu_p \rightarrow min$ is considered, which gives same result.

or in a longer form:

$$\bar{U} = \sum_{n=1}^{n} \omega_i Er_i - \frac{\gamma}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j cov(r_i, r_j) \underset{\omega}{\rightarrow} max \tag{11}$$

where:

γ – risk aversion coefficient, which is positive for an investor characterized by risk aversion (γ > 0).

This coefficient determines the increase in the expected rate of return on a portfolio which should match a one unit increase in risk of the portfolio in order to maintain the unchanged level of the expected portfolio utility. Expected utility is an increasing function of the portfolio's expected rate of return and a decreasing function of the portfolio's risk, measured by the variance of the rate of return.

If we introduce a budget constraint resulting in summing of the portfolio weights to one, which can be written by the formula $w'i = 1$, where i is a column vector of ones. Then we will determine the conditional maximum, and the optimal portfolio structure can be written as follows:

$$\bar{U} = \mu_P - \frac{\gamma}{2} \sigma_p^2 \underset{\omega}{\rightarrow} max \tag{12}$$

with limiting:

$$w'i = 1 \tag{13}$$

Constructing the Lagrange function relevant to the above task:

$$L = \mu_P - \frac{\gamma}{2} \sigma_p^2 - \lambda(w'i - 1) \tag{14}$$

where:

λ – is an indefinite Lagrange multiplier, the vector of optimal weights (w_1), meeting the budget condition, can be written as (Okhrin & Schmid, 2006)[2]:

$$w_1 = \frac{1}{i'\Sigma^{-1}i} \Sigma^{-1} i + \frac{1}{\gamma} \Sigma^{-1} (\mu - \frac{i'\Sigma^{-1}\mu}{i'\Sigma^{-1}i} i) \tag{15}$$

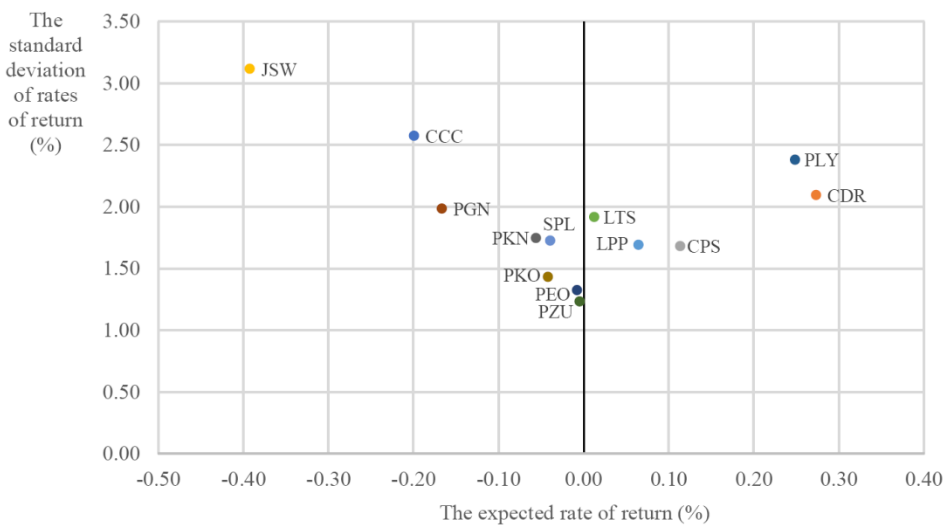
[2] The formulas in Okhrin et al. (2006, p. 237) are written in an equivalent, but slightly different form.

3. Empirical analysis of optimal portfolios

3.1. Selection of company shares for the research

For the research there were selected shares of all companies listed on the Warsaw Stock Exchange included in the WIG 20 index, which in 2019 paid dividends to shareholders. There was a total of 13 such companies (Figure 1, full company names are to be found in Appendix), which represented 65% of all companies from the WIG 20 index. Then, for each company, using the closing prices of shares from all 248 sessions held in 2019, there were calculated 247 daily simple rates of return (excluding the first trading session). During calculations of daily simple rates of return on shares the value of the dividend paid was taken into account by adjusting the reference price (by the value of the dividend paid) at the first trading session without the right to dividend. The expected rate of return and the standard deviation of rates of return were calculated for each company using the values of daily simple rates of return on shares. The results are shown in Figure 1.

Figure 1. The expected values of rates of return and the values of the standard deviation of rates of return on shares of companies for 2019 (%)



Note: Full names of the listed companies are to be found in Appendix.

The analysis of Figure 1 shows that the shares of as many as 8 out of 13 companies included in the WIG 20 index recorded a negative expected rate of return for 2019. The companies controlled by the State Treasury, e.g., JSW and PGN, fared poorly in this respect.

3.2. Construction of portfolios made up of a different number of companies

As it has been shown, the portfolio variance and, as a result, its standard deviation depend not only on the proportion of shares in the portfolio and the variance of their rates of return, but also on the covariance between these rates. Covariance illustrates the relationship between rates of return, however, it does not measure its strength, it only indicates the direction of mutual changes. In other words, it shows to what extent the fluctuations of the rates of return are moving in the same direction. If values of rates of return on shares are high or low in the same period, then the covariance increases and its sign is positive, which indicates a positive relationship between fluctuations in rates of return. Otherwise, i.e., with a negative relationship between the rates of return, the covariance takes values less than zero (Pera et al., 2014). Covariance, however, does not show the strength of the relationship between the variables. Covariance was standardized, i.e., it was divided by the product of standard deviations of the rates of return in order to obtain such an estimate. As a consequence, the Pearson's linear correlation coefficient was obtained (Table 1). Only the calculated coefficient allows to quantify not only the direction but also the strength of the relationship between the rates of return.

Table 1. Values of the Pearson correlation coefficient (linear) between the rates of return on shares of companies

Company	CCC	CDR	CPS	JSW	LPP	LTS	PEO	PGN	PKN	PKO	PLY	PZU	SPL
CCC	1.000												
CDR	0.128	1.000											
CPS	0.240	0.040	1.000										
JSW	0.166	0.121	0.071	1.000									
LPP	0.268	0.126	0.224	0.210	1.000								
LTS	0.117	0.084	0.156	0.149	0.251	1.000							
PEO	0.282	0.125	0.227	0.296	0.296	0.220	1.000						
PGN	0.264	0.232	0.217	0.275	0.417	0.344	0.305	1.000					
PKN	0.137	0.119	0.062	0.244	0.210	0.463	0.313	0.446	1.000				
PKO	0.197	0.154	0.285	0.322	0.329	0.241	0.592	0.340	0.314	1.000			
PLY	–0.015	0.062	0.232	0.020	0.066	0.002	0.062	–0.035	0.053	0.144	1.000		
PZU	0.115	0.137	0.151	0.286	0.258	0.192	0.538	0.267	0.275	0.528	0.079	1.000	
SPL	0.257	0.173	0.177	0.259	0.476	0.241	0.323	0.271	0.201	0.533	0.123	0.418	1.000

As presented in Table 1, the values of the Pearson's correlation coefficient (linear) between pairs of rates of return on shares of companies are all except for two positive. Correlation coefficients greater than zero mean that with the growth of portfolio instruments, the portfolio risk tends to the average level of covariance between the rates of return of shares in the portfolio.

Having obtained Pearson's correlation coefficient between rates of return and the values of expected rate of return, portfolios consisting of a different number of companies were constructed, starting with a portfolio of two companies, and ending with a portfolio of thirteen companies, consisting of all companies selected for the study. There were two groups of such portfolios constructed. The groups differed in the way companies were chosen for the portfolios. In the first group, such companies which had had the lowest average Pearson's correlation coefficient between rates of return were chosen for the portfolio. However, for the second group such companies were chosen for the portfolio which had had the highest average value of expected rates of return. The portfolios constructed in this way are presented in Table 2.

Table 2. Portfolios composed of different numbers of companies

No. of companies in portfolio	Companies in portfolio with the lowest average Pearson's correlation coefficient between rates of return			Companies in portfolio with the highest average value of expected rates of return	
	no. of possible combinations	companies in portfolio	average value of correlation coefficient	companies in portfolio	average value of expected rates of return (%)
2	78	PGN, PLY	−0.0345	CDR, PLY	0.2609
3	286	CCC, LTS, PLY	0.0346	CDR, PLY, CPS	0.2118
4	715	CCC, LTS, PLY, CDR	0.0629	CDR, PLY, CPS, LPP	0.1751
5	1 287	CCC, LTS, PLY, CDR, JSW	0.0833	CDR, PLY, CPS, LPP, LTS	0.1425
6	1 716	CCC, LTS, PLY, CDR, JSW, CPS	0.1048	CDR, PLY, CPS, LPP, LTS, PZU	0.1180
7	1 716	CCC, LTS, PLY, CDR, JSW, CPS, PZU	0.1206	CDR, PLY, CPS, LPP, LTS, PZU, PEO	0.1000
8	1 287	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN	0,1388	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL	0.0826
9	715	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN, LPP	0.1528	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL, PKO	0.0688
10	286	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN, LPP, SPL	0.1738	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL, PKO, PKN	0.0563
11	78	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN, LPP, SPL, PEO	0.1910	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL, PKO, PKN, PGN	0.0360
12	13	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN, LPP, SPL, PEO, PGN	0.2047	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL, PKO, PKN, PGN, CCC	0.0164
13	1	CCC, LTS, PLY, CDR, JSW, CPS, PZU, PKN, LPP, SPL, PEO, PGN, PKO	0.2242	CDR, PLY, CPS, LPP, LTS, PZU, PEO, SPL, PKO, PKN, PGN, CCC, JSW	−0.0150

The data in Table 2 demonstrate that the two-component portfolio, due to the criterion of selecting companies for the portfolio with the lowest average value of the Pearson's correlation coefficient between rates of return is created by companies such as PGN and PLY (which stems directly from Table 1). The number of possible combinations for the two-component portfolio is 78, which is the number of different combinations of pairs of companies that can be created using 13 companies. In order to obtain a portfolio of three companies, the number of combinations increases to 286. The lowest average value of the correlation coefficient at the level of 0.0346 has been recorded for CCC, LTS and PLY. Subsequent portfolios were constructed in a similar way – from the number of possible combinations of companies, the one with the lowest average correlation coefficient was selected. It is also worth mentioning that the average value of the expected rates of return on shares of all 13 companies was negative (−0.015%), while their average value of the Pearson's correlation coefficient was 0.2242, which can be interpreted as a clear, however, low linear relationship.

3.3. Shares (weights) of company equities in optimal portfolios

In this part of the study there were selected portfolio weights maximizing the expected (square) utility function of a portfolio for a given, assumed value of the risk aversion ratio. The risk aversion coefficient was arbitrarily set at levels of $\gamma = 10$ and $\gamma = 100$ because, as the study results prove (Duan, 2007; Farkhati et al., 2014):

1. When $\gamma < 1$ the investor should invest all theirs funds in shares with the highest possible rate of return.
2. There is no significant difference in the share allocation strategy with increase of γ from 100 to 1000.
3. The risk aversion coefficient (γ) is prominent in the range of $1 \leq \gamma \leq 100$.

There were assumed two limiting conditions: a budget constraint and no possibility of rapid sale. Table 3 shows the results for the portfolio to which companies were selected on the basis of the average value of the correlation coefficient between the rates of return.

Table 3. Shares (weights) of company equities in the optimal portfolio to which companies were selected on the basis of the average value of the correlation coefficient

No. of companies in portfolio	Risk aversion coefficient $\gamma = 10$						Risk aversion coefficient $\gamma = 100$					
2	1. PGN	[0.3776]	2. PLY	[0.6224]			1. PGN	[0.5660]	2. PLY	[0.4340]		
3	1. CCC	[0.0587]	2. LTS	[0.4399]	3. PLY	[0.5014]	1. CCC	[0.2169]	2. LTS	[0.4468]	3. PLY	[0.3363]
4	1. CCC	[0.0000]	2. LTS	[0.1990]	3. PLY	[0.3403]	1. CCC	[0.1387]	2. LTS	[0.3288]	3. PLY	[0.2519]
	4. CDR	[0.4607]					4. CDR	[0.2806]				
5	1. CCC	[0.0000]	2. LTS	[0.1990]	3. PLY	[0.3403]	1. CCC	[0.1271]	2. LTS	[0.3117]	3. PLY	[0.2456]
	4. CDR	[0.4607]	5. JSW	[0.0000]			4. CDR	[0.2685]	5. JSW	[0.0471]		
6	1. CCC	[0.0000]	2. LTS	[0.0849]	3. PLY	[0.2457]	1. CCC	[0.0605]	2. LTS	[0.2279]	3. PLY	[0.1600]
	4. CDR	[0.3981]	5. JSW	[0.0000]	6. CPS	[0.2713]	4. CDR	[0.2328]	5. JSW	[0.0336]	6. CPS	[0.2852]
7	1. CCC	[0.0000]	2. LTS	[0.0588]	3. PLY	[0.2341]	1. CCC	[0.0303]	2. LTS	[0.1279]	3. PLY	[0.1097]
	4. CDR	[0.3770]	5. JSW	[0.0000]	6. CPS	[0.2436]	4. CDR	[0.1547]	5. JSW	[0.0000]	6. CPS	[0.1844]
	7. PZU	[0.0865]					7. PZU	[0.3930]				
8	1. CCC	[0.0000]	2. LTS	[0.0588]	3. PLY	[0.2341]	1. CCC	[0.0244]	2. LTS	[0.0909]	3. PLY	[0.1040]
	4. CDR	[0.3770]	5. JSW	[0.0000]	6. CPS	[0.2436]	4. CDR	[0.1481]	5. JSW	[0.0000]	6. CPS	[0.1859]
	7. PZU	[0.0865]	8. PKN	[0.0000]			7. PZU	[0.3597]	8. PKN	[0.0870]		
9	1. CCC	[0.0000]	2. LTS	[0.0350]	3. PLY	[0.2297]	1. CCC	[0.0099]	2. LTS	[0.0744]	3. PLY	[0.0997]
	4. CDR	[0.3646]	5. JSW	[0.0000]	6. CPS	[0.2176]	4. CDR	[0.1400]	5. JSW	[0.0000]	6. CPS	[0.1681]
	7. PZU	[0.0402]	8. PKN	[0.0000]	9. LPP	[0.1130]	7. PZU	[0.3243]	8. PKN	[0.0783]	9. LPP	[0.1053]
10	1. CCC	[0.0000]	2. LTS	[0.0350]	3. PLY	[0.2297]	1. CCC	[0.0099]	2. LTS	[0.0744]	3. PLY	[0.0997]
	4. CDR	[0.3646]	5. JSW	[0.0000]	6. CPS	[0.2176]	4. CDR	[0.1400]	5. JSW	[0.0000]	6. CPS	[0.1681]
	7. PZU	[0.0402]	8. PKN	[0.0000]	9. LPP	[0.1130]	7. PZU	[0.3243]	8. PKN	[0.0783]	9. LPP	[0.1053]
	10. SPL	[0.0000]					10.SPL	[0.0000]				
11	1. CCC	[0.0000]	2. LTS	[0.0350]	3. PLY	[0.2297]	1. CCC	[0.0000]	2. LTS	[0.0713]	3. PLY	[0.0982]
	4. CDR	[0.3646]	5. JSW	[0.0000]	6. CPS	[0.2176]	4. CDR	[0.1363]	5. JSW	[0.0000]	6. CPS	[0.1549]
	7. PZU	[0.0402]	8. PKN	[0.0000]	9. LPP	[0.1130]	7. PZU	[0.2567]	8. PKN	[0.0629]	9. LPP	[0.0945]
	10.SPL	[0.0000]	11.PEO	[0.0000]			10.SPL	[0.0000]	11.PEO	[0.1253]		
12	1. CCC	[0.0000]	2. LTS	[0.0350]	3. PLY	[0.2297]	1. CCC	[0.0000]	2. LTS	[0.0713]	3. PLY	[0.0982]
	4. CDR	[0.3646]	5. JSW	[0.0000]	6. CPS	[0.2176]	4. CDR	[0.1363]	5. JSW	[0.0000]	6. CPS	[0.1549]
	7. PZU	[0.0402]	8. PKN	[0.0000]	9. LPP	[0.1130]	7. PZU	[0.2567]	8. PKN	[0.0629]	9. LPP	[0.0945]
	10.SPL	[0.0000]	11.PEO	[0.0000]	12.PGN	[0.0000]	10.SPL	[0.0000]	11.PEO	[0.1253]	12 PGN	[0.0000]
13	1. CCC	[0.0000]	2. LTS	[0.0350]	3. PLY	[0.2297]	1. CCC	[0.0000]	2. LTS	[0.0713]	3. PLY	[0.0982]
	4. CDR	[0.3646]	5. JSW	[0.0000]	6. CPS	[0.2176]	4. CDR	[0.1363]	5. JSW	[0.0000]	6. CPS	[0.1549]
	7. PZU	[0.0402]	8. PKN	[0.0000]	9. LPP	[0.1130]	7. PZU	[0.2567]	8. PKN	[0.0629]	9. LPP	[0.0945]
	10.SPL	[0.0000]	11.PEO	[0.0000]	12.PGN	[0.0000]	10.SPL	[0.0000]	11.PEO	[0.1253]	12.PGN	[0.0000]
	13.PKO	[0.0000]					13.PKO	[0.0000]				

Analysis of the portfolio weights shown in Table 3, which maximize the expected utility of the investment, let concluded that as γ increases from 10 to 100, the investor becomes more risk-sensitive and increases portfolio diversification. It is noteworthy that for an investor with a risk aversion of γ = 100, the optimal portfolio of two companies consists of 56.6% PGN shares and 43.4% PLY shares, although the expected rate of return on PLY shares was 0.2486% and was much higher than the expected rate of return on PGN shares of −0.1668%. However, for an investor with a risk aversion of γ = 10, these proportions will be reversed, as more money will be invested in PLY than PGN shares.

Subsequently, the shares of company equities in the optimal portfolio were calculated, to which companies were selected based on the average value of expected rates of return (Table 4).

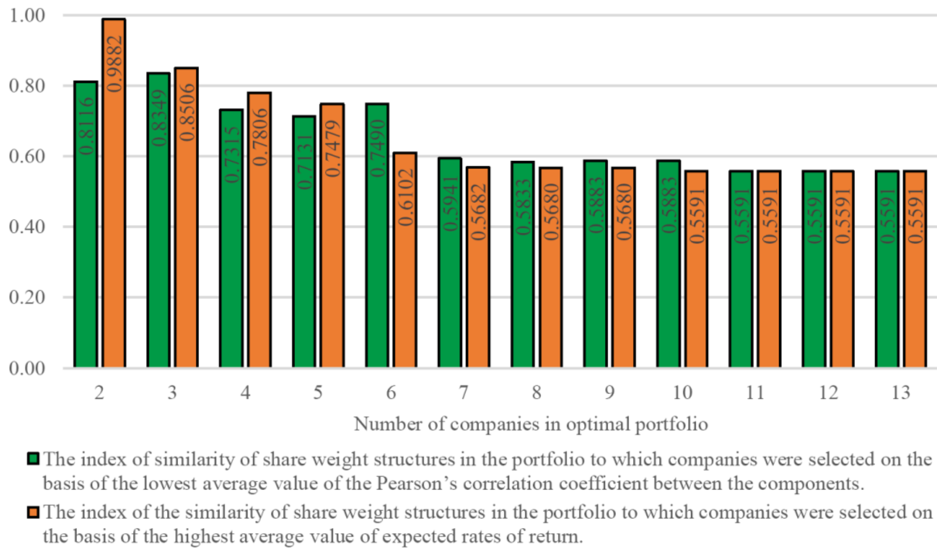
Table 4. Shares (weights) of company equities in the optimal portfolio to which companies were selected on the basis of the average value of expected rates of return

No. of companies in portfolio	Risk aversion coefficient $\gamma = 10$	Risk aversion coefficient $\gamma = 100$
2	1. CDR [0.5806] 2. PLY [0.4194]	1. CDR [0.5688] 2. PLY [0.4312]
3	1. CDR [0.4262] 2. PLY [0.2549] 3. CPS [0.3189]	1. CDR [0.3421] 2. PLY [0.1896] 3. CPS [0.4683]
4	1. CDR [0.3801] 2. PLY [0.2361] 3. CPS [0.2400] 4. LPP [0.1438]	1. CDR [0.2462] 2. PLY [0.1506] 3. CPS [0.3042] 4. LPP [0.2990]
5	1. CDR [0.3719] 2. PLY [0.2340] 3. CPS [0.2263] 4. LPP [0.1247] 5. LTS [0.0432]	1. CDR [0.2122] 2. PLY [0.1415] 3. CPS [0.2469] 4. LPP [0.2195] 5. LTS [0.1800]
6	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402]	1. CDR [0.1476] 2. PLY [0.1036] 3. CPS [0.1708] 4. LPP [0.1165] 5. LTS [0.1074] 6. PZU [0.3542]
7	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000]	1. CDR [0.1404] 2. PLY [0.1016] 3. CPS [0.1521] 4. LPP [0.0987] 5. LTS [0.0960] 6. PZU [0.2698] 7. PEO [0.1415]
8	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000]	1. CDR [0.1404] 2. PLY [0.1016] 3. CPS [0.1521] 4. LPP [0.0987] 5. LTS [0.0960] 6. PZU [0.2698] 7. PEO [0.1415] 8. SPL [0.0000]
9	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000] 9. PKO [0.0000]	1. CDR [0.1404] 2. PLY [0.1016] 3. CPS [0.1521] 4. LPP [0.0987] 5. LTS [0.0960] 6. PZU [0.2698] 7. PEO [0.1415] 8. SPL [0.0000] 9. PKO [0.0000]
10	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0000]	1. CDR [0.1363] 2. PLY [0.0982] 3. CPS [0.1549] 4. LPP [0.0945] 5. LTS [0.0713] 6. PZU [0.2567] 7. PEO [0.1253] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0629]
11	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0000] 11.PGN [0.0000]	1. CDR [0.1363] 2. PLY [0.0982] 3. CPS [0.1549] 4. LPP [0.0945] 5. LTS [0.0713] 6. PZU [0.2567] 7. PEO [0.1253] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0629] 11.PGN [0.0000]
12	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0000] 11.PGN [0.0000] 12.CCC [0.0000]	1. CDR [0.1363] 2. PLY [0.0982] 3. CPS [0.1549] 4. LPP [0.0945] 5. LTS [0.0713] 6. PZU [0.2567] 7. PEO [0.1253] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0629] 11.PGN [0.0000] 12.CCC [0.0000]
13	1. CDR [0.3646] 2. PLY [0.2297] 3. CPS [0.2176] 4. LPP [0.1130] 5. LTS [0.0350] 6. PZU [0.0402] 7. PEO [0.0000] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0000] 11.PGN [0.0000] 12.CCC [0.0000] 13.JSW [0.0000]	1. CDR [0.1363] 2. PLY [0.0982] 3. CPS [0.1549] 4. LPP [0.0945] 5. LTS [0.0713] 6. PZU [0.2567] 7. PEO [0.1253] 8. SPL [0.0000] 9. PKO [0.0000] 10.PKN [0.0629] 11.PGN [0.0000] 12.CCC [0.0000] 13.JSW [0.0000]

In the analysis of the results included in Table 4, it is worth noting that the CDR and PLY companies had the highest average value of the expected rates of return. While in a two-component portfolio, both investors with risk aversion at the level of $\gamma = 10$ and $\gamma = 100$ invest money in similar proportions, then with each increase in the number of companies in the portfolio, the investor characterized by risk aversion at the level of $\gamma = 100$ will invest significantly less money in companies with the highest expected rate of return than the investor with risk aversion at the level of $\gamma = 10$.

The index of structure similarity was calculated in order to test the similarity of the obtained structures of shares of company equities in optimal portfolios for investors characterized by a risk aversion at the level of $\gamma = 10$ level of $\gamma = 100$ (Figure 2).

Figure 2. Index of similarity of share structures (weights) of company equities in optimal portfolios between investors with risk aversion at the level of $\gamma = 10$ level of $\gamma = 100$



The results presented in Figure 2 show, that in optimal portfolios of shares consisting of several (from 2 to 6) companies, the differences in the structure of portfolio weights between the investor characterized by risk at the level of $\gamma = 10$, level of $\gamma = 100$ are smaller than in optimal stock portfolios consisting of more companies (from 7 to 13). It is worth noting that the structure of share weights in a two-component portfolio – consisting of two companies with the highest expected rate of return, for an investor characterized by a risk aversion at the level of

$\gamma = 10$ is almost identical to that for an investor characterized by a risk aversion at the level of $\gamma = 100$. This is obviously related to the fact that the shares of the companies with the highest rate of return are characterized by high risk and the investors who are more sensitive to risk cannot diversify the risk in this case.

3.4. Basic characteristics of constructed optimal portfolios

Having all the necessary data available, for constructed optimal stock portfolios there were calculated the expected rate of return and the standard deviation. A different aversion to risk of investors has been taken into account during the calculations of the above measures (Table 5).

When determining the expected rate of return on a portfolio consisting of shares of 5 companies (according to formula 6), the formula is expressed as follows:

$$\mu_p = \omega_1 \cdot Er_1 + \omega_2 \cdot Er_2 + \omega_3 \cdot Er_3 + \omega_4 \cdot Er_4 + \omega_5 \cdot Er_5$$

while the formula for the portfolio variance is expressed as follows (according to formula 8):

$$\begin{aligned}\sigma_p^2 = {} & \omega_1^2 \cdot \sigma_1^2 + \omega_2^2 \cdot \sigma_3^2 + \omega_3^2 \cdot \sigma_3^2 + \omega_4^2 \cdot \sigma_4^2 + \omega_5^2 \cdot \sigma_5^2 + 2\,(\omega_1 \cdot \sigma_1 \cdot \omega_2 \cdot \sigma_2 \cdot \mathrm{cov}_{12} \\ & + \omega_1 \cdot \sigma_1 \cdot \omega_3 \cdot \sigma_3 \cdot \mathrm{cov}_{13} + \omega_1 \cdot \sigma_1 \cdot \omega_4 \cdot \sigma_4 \cdot \mathrm{cov}_{14} + \omega_1 \cdot \sigma_1 \cdot \omega_5 \cdot \sigma_5 \cdot \mathrm{cov}_{15} \\ & + \omega_2 \cdot \sigma_2 \cdot \omega_3 \cdot \sigma_3 \cdot \mathrm{cov}_{23} + \omega_2 \cdot \sigma_2 \cdot \omega_4 \cdot \sigma_4 \cdot \mathrm{cov}_{24} + \omega_2 \cdot \sigma_2 \cdot \omega_5 \cdot \sigma_5 \cdot \mathrm{cov}_{25} \\ & + \omega_3 \cdot \sigma_3 \cdot \omega_4 \cdot \sigma_4 \cdot \mathrm{cov}_{34} + \omega_3 \cdot \sigma_3 \cdot \omega_5 \cdot \sigma_5 \cdot \mathrm{cov}_{35} + \omega_4 \cdot \sigma_4 \cdot \omega_5 \cdot \sigma_5 \cdot \mathrm{cov}_{45})\end{aligned}$$

Table 5. Expected rate of return on the portfolio and the portfolio standard deviation

No. of companies in portfolio	Portfolio constructed on the basis of the average value of the correlation coefficient				Portfolio constructed on the basis of the average value of expected rates of return			
	risk aversion coefficient $\gamma = 10$		risk aversion coefficient $\gamma = 100$		risk aversion coefficient $\gamma = 10$		risk aversion coefficient $\gamma = 100$	
	rate of return on the portfolio (%)	portfolio standard deviation (%)	rate of return on the portfolio (%)	portfolio standard deviation (%)	rate of return on the portfolio (%)	portfolio standard deviation (%)	rate of return on the portfolio (%)	portfolio standard deviation (%)
2	0.0917	1.6360	0.0135	1.4990	0.2628	1.6202	0.2626	1.6197
3	0.1183	1.4780	0.0459	1.3370	0.2160	1.3070	0.1938	1.2590
4	0.2129	1.3760	0.1157	1.1980	0.1991	1.2110	0.1586	1.1160
5	0.2129	1.3760	0.0944	1.1810	0.1941	1.1870	0.1376	1.0490
6	0.2017	1.2380	0.1133	1.0840	0.1890	**1.1640**	0.0927	0.9091
7	0.1892	1.1790	0.0842	0.9204	0.1890	**1.1640**	0.0861	0.8920
8	0.1892	1.1790	0.0772	0.9069	0.1890	**1.1640**	0.0861	0.8920
9	0.1890	**1.1640**	0.0820	0.8947	0.1890	**1.1640**	0.0861	0.8920
10	0.1890	**1.1640**	0.0820	0.8947	0.1890	**1.1640**	0.0805	**0.8839**
11	0.1890	**1.1640**	0.0805	**0.8839**	0.1890	**1.1640**	0.0805	**0.8839**
12	0.1890	**1.1640**	0.0805	**0.8839**	0.1890	**1.1640**	0.0805	**0.8839**
13	0.1890	**1.1640**	0.0805	**0.8839**	0.1890	**1.1640**	0.0805	**0.8839**

The results in Table 5 let us conclude that as the number of stocks in the portfolio increases, the risk initially decreases and then stabilizes at a certain level. After reaching this level, adding more shares to the portfolio does not result in further risk reduction, as it is indicated in bold. Furthermore, special attention should be paid to a certain homology, namely taking into account the portfolios constructed on the basis of the average value of the expected rates of return, along with the increase in the number of shares in the portfolio, the direction of changes in the value of the expected rate of return and the standard deviation of the portfolio on the basis of the average value of the correlation coefficient, this direction is not the same as the number of shares in the portfolio increases. It can also be seen that the portfolio risk is always lower with an investor's risk aversion $\gamma = 100$ compared to an investor's risk aversion $\gamma = 10$, at the cost of a lower expected return on the portfolio. Furthermore, a certain homology should be considered, where taking into account the portfolios constructed on the basis of the average value of expected rates of return: as the number of shares of companies in the portfolio increases, there is a positive correlation between the direction of changes in the value of the expected rate of return and the standard deviation of the portfolio, whereas, taking into account the portfolios constructed on the basis of the average value of the correlation coefficient, this direction is not the same as the number of shares in the portfolio increases. It can also be noted that the risk of portfolio is always lower with an investor's risk aversion $\gamma = 100$ compared to a risk aversion $\gamma = 10$, at the cost of a lower expected return on the portfolio.

3.5. Ranking of constructed optimal portfolios

In the last part of the study it was sought which of the constructed optimal share portfolios (within particular groups) is the best portfolio, taking into account both the expected rate of return and the portfolio standard deviation. The best, that is having the best relationship between the expected rate of return and the portfolio standard deviation. In an attempt to answer this question, a synthetic measure was used – a taxonomic measure developed by Hellwig (1968), which is a certain function aggregating partial information contained in particular variables, determined for each object[3] (Balcerowicz-Szkutnik & Sojka, 2011).

[3] An object is called an element belonging to a certain set, tested due to certain characteristics (variables).

At the beginning of taxonomic research the nature of the variables was determined, it was therefore assumed that the expected rate of return from the portfolio is the stimulant (a variable for which the higher the value, the better the portfolio qualifies for the research carried out), and the portfolio standard deviation is the destimulant (a variable for which the higher the value, the worse the portfolio qualifies for the research carried out). There was also an assumption made on equal importance of individual variables. The stages of calculating the taxonomic measure of development within individual groups were as follows (Balcerowicz-Szkutnik & Sojka, 2011):

1. The data matrix X was recorded in which any portfolio was marked as x_{ij} $(i = 1, 2, \ldots, 12;\ j = 1, 2)$. This is the value of the *j*-th variable observed in the *i*-th portfolio. The data matrix was then as follows:

$$X = \begin{bmatrix} x_{1\,1} & x_{1\,2} \\ x_{2\,1} & x_{2\,2} \\ \dots & \dots \\ x_{12\,1} & x_{12\,2} \end{bmatrix} \tag{16}$$

2. Variable stimulation – i.e., transforming a destimulant into a stimulant. For this purpose, the quotient transformation as in the formula given was used:

$$x_{ij}^{S} = \max_{i} x_{ij}^{D} - x_{ij}^{D} \tag{17}$$

where:
x_{ij}^{D} – value of *j*-th destimulant in *i*-th portfolio.

3. Carrying out the normalization of variables using standardization method, as in the formula:

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{S\,(x_j)} \tag{18}$$

where:
$\bar{x}_j, S\,(x_j)$ – arithmetic mean and standard deviation of the *j*-th variable.

4. Based on standardized variables there were determined coordinates of the development pattern (the benchmark portfolio with the ‘best’ values for each variable). Initially, the destimulant was converted into a stimulant and therefore, the procedure was based on the selection of the maximum values of standardized variables. It can be written as below:

$$z_0 = [z_{01}, z_{0j}, \ldots z_{0m}] \tag{19}$$

where:
$z_{0j} = \max_{i} z_{ij}$.

5. The Euclidean distance of the portfolio from the established development pattern was calculated with following formula:

$$d_{i0} = \sqrt{\sum_{j=1}^{m} (z_{ij} - z_{0j})^2} \tag{20}$$

where:
d_{i0} – Euclidean distance of i-th portfolio from the pattern of development.

6. For each portfolio a following taxonomic measure of development was calculated:

$$S_i = 1 - \frac{d_{i0}}{d_0} \tag{21}$$

where d_0 was determined with the formula:

$$d_0 = \bar{d} + 2S(d_0) \tag{22}$$

where:
$\bar{d}$ – arithmetic mean of taxonomic distances (d_{i0}),
$S(d_0)$ – standard deviation of taxonomic distances (d_{i0}).

On this basis there was prepared a ranking of constructed optimal stock portfolios, within individual groups. The higher the position in the ranking of an optimal stock portfolio is, the better is the relationship between its expected rate of return and standard deviation. The values of taxonomic measure of development, calculated in accordance with the adopted procedure, are presented in Table 6.

The analysis indicates that the best optimal portfolio in the group of portfolios constructed on the basis of the average value of the correlation coefficient is a portfolio containing shares of six companies, while in the group of portfolios constructed on the basis of average values of expected rates of return, a portfolio containing shares of three companies.

Table 6. Ranking of constructed optimal portfolios

<table>
<tr><td colspan="6">Portfolio constructed on the basis of the average value of the correlation coefficient</td><td colspan="6">Portfolio constructed on the basis of the average value of expected rates of return</td></tr>
<tr><td colspan="3">risk aversion coefficient $\gamma = 10$</td><td colspan="3">risk aversion coefficient $\gamma = 100$</td><td colspan="3">risk aversion coefficient $\gamma = 10$</td><td colspan="3">risk aversion coefficient $\gamma = 100$</td></tr>
<tr><td>position</td><td>no. of companies in portfolio</td><td>value of taxonomic measure of development</td><td>position</td><td>no. of companies in portfolio</td><td>value of taxonomic measure of development</td><td>position</td><td>no. of companies in portfolio</td><td>value of taxonomic measure of development</td><td>position</td><td>no. of companies in portfolio</td><td>value of taxonomic measure of development</td></tr>
<tr><td>1</td><td>6</td><td>0.8485</td><td>1</td><td>6</td><td>0.7563</td><td>1</td><td>3</td><td>0.3731</td><td>1</td><td>3</td><td>0.4466</td></tr>
<tr><td rowspan="5">2</td><td>9</td><td>0.8237</td><td>2</td><td>7</td><td>0.7008</td><td>2</td><td>4</td><td>0.2311</td><td>2</td><td>4</td><td>0.4425</td></tr>
<tr><td>10</td><td>0.8237</td><td rowspan="2">3</td><td>9</td><td>0.6830</td><td>3</td><td>5</td><td>0.1755</td><td>3</td><td>5</td><td>0.3873</td></tr>
<tr><td>11</td><td>0.8237</td><td>10</td><td>0.6830</td><td rowspan="8">4</td><td>6</td><td>0.1156</td><td>4</td><td>6</td><td>0.2117</td></tr>
<tr><td>12</td><td>0.8237</td><td rowspan="3">5</td><td>11</td><td>0.6696</td><td>7</td><td>0.1156</td><td rowspan="3">5</td><td>7</td><td>0.1816</td></tr>
<tr><td>13</td><td>0.8237</td><td>12</td><td>0.6696</td><td>8</td><td>0.1156</td><td>8</td><td>0.1816</td></tr>
<tr><td rowspan="2">7</td><td>7</td><td>0.8233</td><td>13</td><td>0.6696</td><td>9</td><td>0.1156</td><td>9</td><td>0.1816</td></tr>
<tr><td>8</td><td>0.8233</td><td>8</td><td>8</td><td>0.6367</td><td>10</td><td>0.1156</td><td rowspan="4">8</td><td>10</td><td>0.1560</td></tr>
<tr><td rowspan="2">9</td><td>4</td><td>0.6361</td><td>9</td><td>4</td><td>0.6191</td><td>11</td><td>0.1156</td><td>11</td><td>0.1560</td></tr>
<tr><td>5</td><td>0.6361</td><td>10</td><td>5</td><td>0.5880</td><td>12</td><td>0.1156</td><td>12</td><td>0.1560</td></tr>
<tr><td>11</td><td>3</td><td>0.1183</td><td>11</td><td>3</td><td>0.1443</td><td>13</td><td>0.1156</td><td>13</td><td>0.1560</td></tr>
<tr><td>12</td><td>2</td><td>-0.2063</td><td>12</td><td>2</td><td>-0.2160</td><td>12</td><td>2</td><td>0.1065</td><td>12</td><td>2</td><td>0.1130</td></tr>
</table>

The Spearman's correlation coefficient was calculated in order to be able to assess the similarity of the rankings obtained between the investor's risk aversion $\gamma = 10$ and the investor's risk aversion $\gamma = 100$. The calculated value of the correlation measure for the group of portfolios constructed on the basis of the average value of the correlation coefficient is 0.9888, whereas in the group of portfolios constructed on the basis of the average value of the expected rates of return, it is 0.8657. This provides evidence that the obtained rankings are very similar.

4. Discussion

Creating an investment portfolio and, in particular, composing its appropriate content is the most difficult skill in the process of investing capital. This is the way of creating investor's investment portfolio which has a significant impact on the investment results. The study presents selected methods of constructing a portfolio consisting of a different number of companies. Then, portfolio weights were selected to maximize its expected utility, and next optimal portfolios with the best relationship between expected rates of return and the standard deviations were identified.

It should be noted that in the study special attention is deserved by the γ parameter, which characterizes the investor's risk propensity. The designated portfolio weights that maximize the expected (quadratic) portfolio utility function are inversely proportional to the adopted risk-aversion factor γ. The investor buys, depending on its value, a larger or smaller number of company shares for the portfolio.

The conducted empirical research shows that:

1. As the investors' risk aversion increases from $\gamma = 10$ to $\gamma = 100$, the investors increase diversification of their portfolios to a greater extent.
2. The structures of share weights in the optimal portfolios between an investor of a risk aversion of $\gamma = 10$ and an investor of a risk aversion of $\gamma = 100$ are ranging from moderately similar to very similar.
3. The optimal portfolio risk is always lower with an investor's risk aversion of $\gamma = 100$ compared to a risk aversion investor of $\gamma = 10$, although at the cost of a lower expected rate of return.
4. Rankings of optimal portfolios created due to the dependence between the expected rate of return and the standard deviation of the portfolio for the investors of risk aversion $\gamma = 10$ and the risk aversion of $\gamma = 100$ are very similar.

5. Conclusions

Following the literature, this study contributes to the search for the best portfolio among optimal portfolios determined on the basis of the criterion of maximizing the expected (quadratic) function of investment utility, on the example of companies listed on the Warsaw Stock Exchange.

Comparing the weights obtained in optimal equity portfolios, depending on the risk aversion coefficient, with the results described in the source literature, it can be concluded that there are slight differences. Namely, the coefficient of similarity of structures between an investor with risk aversion at the level of $\gamma = 10$ and an investor with risk aversion at the level of $\gamma = 100$ for a portfolio consisting of shares of five companies is 0.7131 and 0.7479, while in the study by Farkhati et al. (2014) it is 0.8329 and in the study by Duan (2007) it is around 0.86, while for a portfolio consisting of shares of 12 companies it is 0.5591, while in the study conducted by Septiano et al. (2019) is 0.6615. This results from the fact that the differences in weight structures in optimal equity portfolios between investors with risk aversion at the level of $\gamma = 10$ and $\gamma = 100$ in this study are greater than in the studies of the above-mentioned authors. It is also worth adding that in all the studies conducted, regardless of whether it was a portfolio consisting of five or twelve companies, an investor with a risk aversion at the level of $\gamma = 10$ always invests more money in a company with the highest rate of return than an investor with a risk aversion at the level of $\gamma = 100$.

In practice of a stockbroker, a volatility coefficient is often used, which determines the portfolio risk per unit of expected rate of return. The disadvantage of this ratio is that it can only be used for a positive rate of return. If, by means of this ratio, one wanted to compare portfolios on the basis of the principle of profit maximization and risk minimization, and the rate of return was negative, the ratio would also be less than zero and it would prefer negative portfolios to portfolios with a positive rate of return. In this study, a synthetic variable was proposed for the comparison of portfolios – the measure of development by Hellwig (1968). With this measure, the diagnostic variables underlying the construction of the synthetic variable may take negative values. Moreover, the vector of diagnostic variables does not have to consist only of the expected rate of return and risk, but may include other variables, in addition, these variables can be assigned different weights depending on the investor's preferences.

The research limitation is the fact that the study covers only one year. Thus, subsequent studies might use data from other years and compare the obtained results, determining whether the strategies presented are appropriate. It is also possible, on the basis of other criteria, to construct portfolios consisting of a different number of companies. This means, for example, the selection of companies for a portfolio based on their risk or on the basis of a non-linear measure of correlation.

Appendix

Table A. The list of companies selected for the study

No.	The company name	Ticker/stock symbol
1.	Bank Polska Kasa Opieki Spółka Akcyjna	PEO
2.	CCC Spółka Akcyjna	CCC
3.	CD Projekt Spółka Akcyjna	CDR
4.	Cyfrowy Polsat Spółka Akcyjna	CPS
5.	Grupa Lotos Spółka Akcyjna	LTS
6.	Jastrzębska Spółka Węglowa Spółka Akcyjna	JSW
7.	LPP Spółka Akcyjna	LPP
8.	Play Communications Societe Anonyme	PLY
9.	Polski Koncern Naftowy Orlen Spółka Akcyjna	PKN
10.	Polskie Górnictwo Naftowe i Gazownictwo Spółka Akcyjna	PGN
11.	Powszechna Kasa Oszczędności Bank Polski	PKO
12.	Powszechny Zakład Ubezpieczeń Spółka Akcyjna	PZU
13.	Santander Bank Polska Spółka Akcyjna	SPL

Source: Based on: data from the Warsaw Stock Exchange (n.d.).

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