Abstract

The paper relates to the trading systems supporting traders making decision on the forex market. Typical trading systems using technical analysis generate a buy or sell signal when the technical indicator crosses a given oversell or overbought levels. The paper extends the approach in which the above strict crisp conditions are replaced by fuzzy relations. The indicators are treated not independently as it is in the typical systems but jointly. Currency pairs are compared in the multicriteria space in which each criterion is defined by a membership function referring to a given indicator. New formulations of the membership functions for different indicators are proposed. General ideas of the algorithm generating non-dominated alternatives in the multicriteria space are presented. The algorithm has been implemented in an experimental system. Computational results for different time windows using real-world data from the forex market are presented and discussed.

Keywords: multicriteria decision making, trading systems, fuzzy sets.
1 Introduction

Fully automatic trading systems along with various social trading platforms are nowadays becoming one of the most popular possibilities to invest in a range number of markets. A still growing number of instruments available to invest creates a wide range of opportunities to create Intermarket portfolios. At the same time, the high accessibility of various financial analysis tools opens possibilities for decision makers to create their own strategies. Unfortunately, the lack of transparency of risk-related concepts often leads to a situation in which many investors use very risky strategies without any prior domain knowledge. This is especially true for the high-volatility markets related to currency pairs (such as forex market) and cryptocurrencies. Extreme volatility, and at the same time little predictability of possible crucial events on the market can easily lead to losses in the account balance.

The concepts presented here can be adapted to any market, but, due to the large variety of instruments, correlations among instruments and high liquidity on the market, we select forex as our experimental environment. Among some indisputable advantages of the forex market, we could mention the very low transactions costs. This supports the approach that this element of additional costs can be omitted. Thus this paper is entirely directed towards the efficiency of trading systems itself. More information about the forex market with emphasis on trading systems can be found in Chaboud et al. (2014).

Tools present on various trading platforms can be successfully adapted to various instruments. For some more liquid markets, however, it is more natural that such an environment attracts more attention from the decision makers. In the sense of financial analysis, we can note the two most popular concepts used as an order-opening trigger on the market. The first one is called fundamental analysis and includes the use of various economic factors that have a great potential impact on the price direction. The second concept emerges from the Dow theory and is called technical analysis. Assuming that history repeats itself, the various mathematical formulas are in this case used to calculate market indicator values. Finally, these indicators are analyzed to seek patterns and similarities. With the use of some earlier predefined rules we assume that the behavior of the market will be repeated.

Unfortunately, to seek very promising signals, a number of different indicators should be analyzed at the same time. That obviously leads to a very small number of signals generated directly to the decision maker. The existing crisp trading systems based on certain binary rules rarely give signals.
On the other hand, simple trading systems focused on a few indicators generate a large number of poor quality signals.

This paper presents current results of the research dealing with multicriteria fuzzy approach supporting decisions on the forex market. The initial results of the research have been discussed in Juszczyk and Kruś (2017). New results in this paper include the implementation of three classes of trading systems: traditional (crisp), fuzzy and multicriteria fuzzy. A new formulation of the fuzzy activation function (membership function), general for different indicators is proposed and applied in the second and third classes of the systems. In the third class, the dominance-based algorithm generating non-dominated alternatives is used. Numerical experiments on real-world data from the forex market have been performed for different sets of indicators using the implemented systems. The efficiency of the signals generated by the systems is derived and compared.

This paper is organized as follows. In the next section, we give a short review of the most often cited papers related to trading systems and fuzzy trading systems on the forex market. Section 3 describes in detail a crucial aspect of the proposed approach, that is, the concept of the fuzzification for the selected rules, and also recalls the algorithm used for the selection of non-dominated alternatives. Section 4 describes three cases related to the differences between crisp and fuzzy multicriteria approaches. Section 5 includes numerical experiments, while section 6 consists of conclusions.

2 Related research

The fully automatic trading system is a concept closely related to high-frequency trading. Despite the approach selected (fully automatic or decision support), the forex market seems a perfect fit for a rule-based trading system. One of the important advantages on the market are low transaction costs and the possibility to use the leverage. This supports the approach for which the system could be effective achieving at least a small advantage over random trades (Sewell and Yan, 2008).

Systems including various elements of machine learning have been always commonly used as an investing tool. Approaches related to support vector machines (Lu and Wu, 2009) or widely discussed neural network methods (Kamruzzaman and Sarker, 2003) are commonly used and can be very efficient. One of the most recent papers discussing the use of neural networks along with the gene expression approach can be found in Sermpinis et al. (2012). The main drawback of these methods, however, is their lack of detailed explanation of the signal and of possibility to use this knowledge
in the future. The so-called “black-box” approach derives only the result, while the question “how” the signal was derived remains unresolved.

Applications of fuzzy sets to the market are not new. There have been attempts to derive an effective trading system based on the fuzzy sets theory. Some of the first concepts based on Mamdani’s rules were introduced in Dourra and Siy (2002). A similar concept assuming the use of rough sets theory was presented in Wang (2003). On the other hand, there is still a wide range of papers proposing new trading rules. For example, Neely et al. (1997) focuses on deriving the new rules using genetic programming. In Neely and Weller (1999) the authors use the same method to generate rules on the basis of a 10-years time span.

Certain machine learning methods adapted for use in forex trading can be found in Booth et al. (2014) whose authors proposed a concept based on the random forest approach. An interesting approach for deriving a support decision system based on the wisdom of the crowd was presented in Gottschlich and Hinz (2014). The authors used the crowd’s recommendations as an element of the investment decision. First experiments have shown that this approach could outperform the market benchmark.

A trading system with elements of fundamental analysis was presented in Nassirtoussi et al. (2015). In their analysis, the authors applied text-mining techniques to estimate the direction of daily currency pair changes using news headlines.

All the methods described above apply different approaches combining concepts related to fuzzy sets, decision support systems, automatic rule generation, machine learning techniques, etc. Our proposed approach takes the best concepts from crisp trading systems and expands them so that more signals can be included. The proposed fuzzy trading systems offer more opportunities to open the transactions without significant decrease of the quality of the signals.

3 The development of fuzzified trading systems

Any trading system: fully automatic or a decision support system, can be represented as a flow between independent modules. In this section, we present crucial aspects of our approach. Among these elements, we can find the set of rules related to different market indicators, the process of the fuzzification of criteria and, finally, the dominance-based approach to the derivation of the set of solutions for the decision maker. We developed a system which also allows for a simple comparison of crisp, fuzzy and
multicriteria fuzzy approaches described further in this section. The general schema of such systems can be found in Figure 1.

![General schema of the trading system](image)

**Figure 1: General schema of the trading system**

Please note that the entire paper is devoted to deriving the phase responsible for the derivation of the signals. We have also implemented in detail the last phase of a system which allows for the initial evaluation of the signals derived.

### 3.1 Trading rules and the proposed fuzzy approach

Technical analysis includes dozens of different indicators based on different rules and assumptions. In this paper, we have selected a few indicators most commonly used in modern financial analysis. It is important to note that the concept of fuzzification presented further in this section can be easily adapted to different indicators as well. Additionally, we focus on BUY signals, but opposite rules can be adapted to SELL signals as well.

The general trading rule for all indicators considered in this research is as follows:

\[
cond_{ind_{Buy}} = \text{true} \text{ if } (ind_n(t-1) < c) \land (ind_n(t) > c),
\]

where \( ind \) is any of the indicators used in our research, \( t - 1 \) and \( t \) are two successive, discrete time readings, while \( c \) is the threshold value for the indicator to generate the signal. The \( c \) values for different indicators, in the traditional (crisp) systems, are as follows:

- CCI (Commodity Channel Index), threshold value equal to \(-100\);
- RSI (Relative Strength Index), threshold value equal to \(0.3\);
- DM (DeMarker), threshold value equal to \(0.3\);
• Stoch (Stochastic oscillator), threshold value equal to 20 – for the main signal line.

Several readings taken to calculate the values of the indicators were the default and equal to 14. More on the example rules and technical analysis indicators can be found, for example, in Kirkpatrick and Dahlquist (2010).

In the traditional (crisp) trading system the signal is generated only when the indicator value increases and crosses the strict threshold value $c$ according to rule (1). Otherwise, the signal is not generated. In the proposed fuzzification we assume that the threshold value is fuzzy. This means that a fuzzy signal can be generated also when the indicator value crosses a threshold lower or higher than $c$. In such a case the strength of the signal is lower than in the case of the crisp system.

The most important assumption in the fuzzification process, as in crisp systems, is the dependency between two successive indicator values. The process is conducted only when $ind(t) > ind(t - 1)$, thus $\Delta ind > 0$, where $\Delta ind = ind(t) - ind(t - 1)$. The strength of the fuzzy signal is calculated by a fuzzy activation function, that is, a membership function with values from $[0, 1]$. It takes value 1 when the crisp threshold $c$ is crossed. Otherwise, it takes values below 1. One should note that the original signal generated by the crisp version is always present, while the proposed fuzzy approach extends the possibility of the occurrence of additional signals.

Figure 2 shows the example fuzzification process for the CCI indicator. The upper part of the window includes the original CCI data. The indicator value used in the crisp rules is described as “crisp” on the right-hand side of the chart. Two additional boundaries labelled as “upper bound” and “lower bound” are used to set the maximal and minimal indicator values, for which the fuzzy activation function receives a value different from 0. Black vertical lines seen on the upper chart show those fragments of the chart which will be transformed into non-zero values of the fuzzy activation function.

The lower part of Figure 2 corresponds to the generated fuzzy activation functions, which are further used in the experiments.

To sum up:

• in the crisp approach, the binary activation function is present, and possible criterion (market indicator) values are equal to 0 or 1;

• in the fuzzy approach, the initial indicator values are transformed into the fuzzy activation function, for which non-zero values are observed only if the difference between the second and the first indicator values is positive, and at the same time the indicator is within the range $\langle lowerbound; upperbound \rangle$. 

3.2 Dominance based-algorithm

The traditional (crisp) systems and the proposed fuzzy system generate several signals independently for different currency pairs and different indicators. In general, the systems do not provide information about relations between the signals which would allow to select the best (that is, the most promising) alternatives to make a decision on the market. Therefore, a multicriteria optimization approach is proposed. In this approach, each currency pair is treated as an alternative evaluated by a vector of criteria related to different indicators. The value of each criterion is defined by the fuzzy activation function of the respective indicator and takes a value from the interval \(0, 1\). All the currency pairs for which the fuzzy signals are generated can be analyzed and compared in the multicriteria space. A multicriteria optimization problem can be formulated in which we look for non-dominated alternatives. Thus we use the concept originally introduced by the authors of Juszczuk and Kruś (2017). The proposed algorithm which uses dominance relations generates a set of non-dominated alternatives for the decision maker. The concept of the reservation point \(x\) is used to initially exclude certain alternatives which do not satisfy the minimal requirements derived by the decision maker.

Input parameters for the dominance-based algorithms are as follows:

- the initial position of the reservation point \(x\);
- the full set of alternatives to be analyzed, denoted by \(Y\);
- the set \(Y_\sim\), which will include all the alternatives removed by the algorithm;
- the set \(Y_+\), which includes alternatives non-dominated by the point \(x\) and analyzed further in the algorithm;
- the set \(ND\), which will include all the non-dominated alternatives selected from the set \(Y_+\).
The whole procedure can be divided into three separate phases. In the first phase, we search for alternatives equal to the aspiration point \( u \) (the point for which all criteria take value 1), which could result in an immediate termination of the algorithm and in the derivation of such an alternative by the decision maker. This phase is represented in lines 3-4. The second, longest phase corresponds to the sequential analysis of all alternatives from the set \( Y_+ \). All non-dominated alternatives will be moved to the set \( ND \). This phase is represented in lines 5-19. Finally, when there are no more alternatives left to analyze in \( Y_+ \), the set \( ND \) is given as well as the number of signals generated according to this set.

To calculate the quality of the derived signals we introduce an efficiency measure based on the accuracy formula:

\[
acc(p, \epsilon) = \frac{TP}{FP + TP},
\]

where \( TP \) is the number of all the signals for which after \( p \) readings we have observed at least an \( \epsilon \) price rise, while \( FP \) is the number of all other signals. In general, the efficiency of a trading system is measured as a ratio of the number of acceptable signals to all signals.

4 Case analysis

In this section, we present analysis, discussion and explanation of three different possible cases that could occur on the market in the crisp, fuzzy, and multicriteria approaches.

4.1 Case 1: the existence of alternatives equal to the aspiration point

The existence of alternatives equal to the aspiration point \( u \) in the crisp version as well as in the fuzzy version leads to an immediate derivation of the set \( ND \) by the algorithm for the decision maker. Each element of the set dominates all remaining alternatives. This situation is presented in Figure 3. Alternative \( y_2 = u \) is selected by the algorithm and derived for the decision maker. It dominates all other alternatives. It is selected from all the alternatives generated by the fuzzy system, shown on the left-hand side of the figure. The crisp system generates signals shown on the right-hand side of the figure.
Algorithm 1: Dominance-based algorithm

begin
1    Fix the aspiration point $u$, create sets $Y$ and $ND = \emptyset$
2    Set the point $x$ and derive sets $Y_-$ and $Y_+$
3    if there exists $y \in Y$ such that $y = u$ then
4        $ND = \{y\}$ End of the algorithm
5    for each alternative $y$ in $Y_+$ do
6        if $y \in Y_-$ then
7            Delete $y$ from further analysis, i.e. $Y_+ = Y_+ \setminus \{y\}$
8        else if $y \not\in Y_- \land ND = \emptyset$ then
9            Add $y$ to set $ND$ and Update sets $Y_-$ and $Y_+ = Y_+ \setminus \{y\}$.
10       else
11          for $z \in ND$ do
12            if $y \succ z$ then
13                Delete $z$ from $ND$
14            else if $z \succ y$ then
15                Mark $y$ as dominated, delete it from $Y_+$, and
16                BREAK
17          if $y$ is non-dominated then
18                Add $y$ to $ND$
19                Update set $Y_- = Y_- \cup (y + \mathbb{R}_+^2 \setminus \{0\})$
20                Delete $y$ from further analysis, i.e. $Y_+ = Y_+ \setminus \{y\}$
21          if $Y_+ = \emptyset$ then
22                End of the algorithm
end
4.2 Case 2: the existence of dominated alternatives on the boundaries of the search space

The crisp approach does not allow to differentiate the alternatives observed within the boundaries of the search space. Thus every such alternative will be treated as a possible solution derived for the decision maker. This situation for the crisp approach is shown in Figure 4. In this example, the fuzzy system generates four alternatives. Two non-dominated alternatives \( y_2 \) and \( y_3 \) are selected by the algorithm. The traditional crisp system generates four signals (the right-hand side of the figure) without providing information as to which one is better or worse.
At the same time the fuzzy approach proposed here allows to differentiate these alternatives and non-dominated alternatives. The number of such alternatives for the boundary case will be no greater than \(n - 1\), where \(n\) is the number of criteria available in the system. In general in this case the dominance-based algorithm derives a lower number of non-dominated alternatives than the number of signals generated by the crisp system.

### 4.3 Case 3: the lack of alternatives on the boundaries of the search space

This case describes the situation for which none of the market indicators present in the system allows in the crisp system to derive a single alternative observed on the boundaries of the search space. The fuzzy approach allows to indicate the alternatives which are relatively close to the boundaries of the search space, where \(\text{crit}_i < 1\) for any \(i\). By setting the position of the reservation point \(x\), the decision maker can adjust the potential risk related to the situation, by eliminating all alternatives worse than \(x\).

The boundary situation for this case is \(x = u\), where no alternatives other than one equal to the aspiration point are good enough for the decision maker. Moving \(x\) towards \(u\) expands the search space in which alternatives of potential interest for the decision maker can be found. The second boundary situation, in which \(x = 0\), is the situation in which all non-dominated alternatives present in the set of alternatives are derived for the decision maker. This situation is shown in Figure 5.

![Figure 5: a) Step s^2 of the algorithm; b) the crisp system does not generate signals](image-url)

To sum up, for the crisp approach the possible signals are generated only when at least one of the criteria considered is equal to 1. For the proposed fuzzy system, this case is extended for the situation in which the value of at least one criterion is greater than \(x\). In general, the fuzzy approach includes all signals generated by the crisp approach and also additional signals, for which \(\exists \text{crit}_i, x < \text{crit}_i < u, i = 1, 2, ..., n\), where \(n\) is the number of criteria.
considered. Finally, in the proposed multicriteria fuzzy approach we assume that dominance relations in the criteria space are included. Thus only non-dominated alternatives from the previous fuzzy approach are selected and derived for the decision maker.

5 Numerical experiments

In this section, we perform a set of experiments using real-world data. To estimate the efficiency of different systems and different trades we analyzed 15 different currency pairs and a time span equal to 16 months (from September 2016 to December 2017). All experiments were performed on 10 different trading systems and three different approaches: crisp, fuzzy, and multicriteria fuzzy approaches. We selected all possible combinations of indicators described in section 2 (with at least two indicators). The position of the reservation point $x$ for all cases was set to 0.85. Below we discuss selected results of these systems:

- The number of signals derived by each trading system and each approach,
- The efficiency of the trading system,
- The efficiency of the trading system in which only alternatives equal to $u$ were analyzed.

First of all, in Table 1, we analyzed the number of signals generated by the systems. The crisp approach is used as a benchmark for the proposed fuzzy approaches. For the crisp approach, we assumed that at least one criterion should be equal to 1, so the ideal alternatives (equal to point $u$) are included in this comparison as well.

A similar assumption was used for our proposed fuzzy approach. However, instead of the classical binary activation functions, fuzzy activation functions were used. It is obvious that the number of signals generated will be slightly higher than the numbers included for the crisp approach; compare with Case 3 in section 4.3.

Finally, to limit the number of signals generated for the decision maker and to improve their efficiency, we applied our proposed dominance-based algorithm, which was used to derive only non-dominated signals. This number is shown in the last column of Table 1. We note the limited number of signals where the dominance concept is adapted.

In Table 2 we analyzed the average accuracy for all the trading systems and three different approaches. The accuracy for all cases was measured
after 3 readings \((p = 3)\). The minimal price difference between the actual price and the price observed 3 readings ago was equal to \(\epsilon = 10\). Note that we did not use any additional money management method, thus we focus mostly on comparing results for different approaches. Thus, as it was expected, the accuracy is fairly small.

Note that despite a larger number of signals derived by the fuzzy approach (as compared with the crisp approach), the accuracy dropped only for certain selected currency pairs. That implies that small deviations from the crisp approach as well as moving the reservation point \(x\) away from the aspiration point \(u\) does not necessarily mean a significant drop in accuracy.

At the same time, the results derived for the MCDM fuzzy approach are better than those for the two remaining cases. This is mostly due to the fact that only non-dominated solutions have been included. However, there is still room for improvements, because there was no trading system which could achieve the accuracy value above 50%. The results are presented as the average for all currency pairs, so we could still assume that for the specific instruments threshold of 50% could be broken.

Finally, in Table 3 we present the accuracy for all the alternatives equal to the aspiration point \(u\). This is obviously the same for all the approaches, so the results are included in one column only. The most important observation is that the alternatives for which all the criteria are equal to 1 provide better results. For most of the trading systems we observed an accuracy above 50%. At the same time, one should note that for the MCDM fuzzy approach the number of market indicators involved in the trading system does not significantly affect the accuracy. However, it obviously affects the number of signals generated for the decision maker.

### Table 1: Number of signals derived by all systems for three different approaches

<table>
<thead>
<tr>
<th>Trading system</th>
<th>Crisp</th>
<th>Fuzzy</th>
<th>MCDM Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCI RSI Stoch DM</td>
<td>4126</td>
<td>4354</td>
<td>2713</td>
</tr>
<tr>
<td>CCI RSI Stoch</td>
<td>3417</td>
<td>3643</td>
<td>2150</td>
</tr>
<tr>
<td>CCI RSI DM</td>
<td>3112</td>
<td>3249</td>
<td>2043</td>
</tr>
<tr>
<td>RSI Stoch DM</td>
<td>2610</td>
<td>3013</td>
<td>1875</td>
</tr>
<tr>
<td>CCI RSI</td>
<td>2334</td>
<td>2405</td>
<td>1442</td>
</tr>
<tr>
<td>CCI Stoch</td>
<td>3165</td>
<td>3349</td>
<td>1934</td>
</tr>
<tr>
<td>CCI DM</td>
<td>2853</td>
<td>3130</td>
<td>1806</td>
</tr>
<tr>
<td>RSI Stoch</td>
<td>1753</td>
<td>1934</td>
<td>1226</td>
</tr>
<tr>
<td>RSI DM</td>
<td>1387</td>
<td>1458</td>
<td>1075</td>
</tr>
<tr>
<td>Stoch DM</td>
<td>2320</td>
<td>2504</td>
<td>1637</td>
</tr>
</tbody>
</table>
Table 2: Accuracy of different trading systems for the three approaches analyzed

<table>
<thead>
<tr>
<th>Trading system</th>
<th>Crisp</th>
<th>Fuzzy</th>
<th>MCDM Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCI RSI Stoch DM</td>
<td>46.04%</td>
<td>43.04%</td>
<td>48.34%</td>
</tr>
<tr>
<td>CCI RSI Stoch</td>
<td>43.68%</td>
<td>45.68%</td>
<td>47.9%</td>
</tr>
<tr>
<td>CCI RSI DM</td>
<td>42.27%</td>
<td>43.27%</td>
<td>48.51%</td>
</tr>
<tr>
<td>RSI Stoch DM</td>
<td>49.13%</td>
<td>43.18%</td>
<td>49.15%</td>
</tr>
<tr>
<td>CCI RSI</td>
<td>48.20%</td>
<td>48.20%</td>
<td>48.58%</td>
</tr>
<tr>
<td>CCI Stoch</td>
<td>41.96%</td>
<td>43.96%</td>
<td>48.00%</td>
</tr>
<tr>
<td>CCI DM</td>
<td>44.31%</td>
<td>45.31%</td>
<td>48.36%</td>
</tr>
<tr>
<td>RSI Stoch</td>
<td>42.71%</td>
<td>40.71%</td>
<td>47.83%</td>
</tr>
<tr>
<td>RSI DM</td>
<td>45.96%</td>
<td>44.96%</td>
<td>49.16%</td>
</tr>
<tr>
<td>Stoch DM</td>
<td>49.11%</td>
<td>49.11%</td>
<td>48.78%</td>
</tr>
</tbody>
</table>

Table 3: Accuracy for the case in which an alternative equal to aspiration point \( u \) exists

<table>
<thead>
<tr>
<th>Trading System</th>
<th>Signals</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCI RSI Stoch DM</td>
<td>3</td>
<td>66.67%</td>
</tr>
<tr>
<td>CCI RSI Stoch</td>
<td>16</td>
<td>81.25%</td>
</tr>
<tr>
<td>CCI RSI DM</td>
<td>20</td>
<td>50.00%</td>
</tr>
<tr>
<td>RSI Stoch DM</td>
<td>11</td>
<td>54.55%</td>
</tr>
<tr>
<td>CCI RSI</td>
<td>122</td>
<td>52.86%</td>
</tr>
<tr>
<td>CCI Stoch</td>
<td>246</td>
<td>44.87%</td>
</tr>
<tr>
<td>CCI DM</td>
<td>189</td>
<td>60.23%</td>
</tr>
<tr>
<td>RSI Stoch</td>
<td>82</td>
<td>53.85%</td>
</tr>
<tr>
<td>RSI DM</td>
<td>79</td>
<td>44.67%</td>
</tr>
<tr>
<td>Stoch DM</td>
<td>101</td>
<td>53.00%</td>
</tr>
</tbody>
</table>

6 Conclusions

In this paper, we have described the concept of fuzzy trading systems for the forex market. The original crisp trading system involving the use of crisp rules has been transformed into the fuzzy approach. Each criterion present in the crisp version of the system has been changed to corresponded to the concept of the single fuzzy membership function. Next, the rank-domination based algorithm has been discussed. This method allowed to derive a set of non-dominated alternatives which can be further presented to the decision maker.

In the experimental section of this paper, we have studied three different cases related to the position of alternatives for different trading systems (crisp, fuzzy, and multicriteria fuzzy approaches). The results of the observations were confirmed in the numerical section of the paper, where
we analyzed the number of alternatives derived for the user by all three approaches. The efficiency of different indicators was measured using the classical accuracy measure. Finally, the efficiency of the ideal alternatives was verified and estimated.

This research shows a simple transition from the classical crisp trading systems to the multicriteria approach with the use of fuzzy sets.

References


