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FUZZY TOPSIS METHOD FOR GROUP DECISION MAKING

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Abstract

Multiple Criteria Decision Making methods have become very popular in recent years and are frequently applied in many real-life situations. The increasing complexity of the decision problems analysed makes it less feasible to consider all the relevant aspects of the problems by a single decision maker. As a result, many real-life problems are discussed by a group of decision makers.

The aim of the paper is to present a new approach for ranking of alternatives with fuzzy data for group decision making using the TOPSIS method. In the proposed approach, all individual decision information of decision makers is taken into account in determining the ranking of alternatives and selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. A numerical example illustrates the proposed approach.

Keywords: fuzzy numbers, TOPSIS, group decision making, aggregation fuzzy numbers.

1 Introduction

Multiple Criteria Decision Making (MCDM) methods have become very popular in recent years and are frequently applied in many real-life situations (for more information see, e.g., Behzadian et al., 2012; Abdullah and Adawiyah, 2014). One of the most popular and widely applied MCDM methods is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang and Yoon (1981). The basic idea of this method is fairly straightforward. It uses two reference points: the so-called positive ideal solution (PIS) and negative ideal solution (NIS) as benchmarks. The chosen alternative is that

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which has both the shortest distance from the PIS and the longest distance from the NIS. The PIS is a solution that maximizes all the benefit criteria and minimizes all the cost criteria, whereas the NIS is a solution that maximizes all the cost criteria and minimizes all the benefit criteria.

The classical TOPSIS method is based on the information provided by the decision maker (DM) or expert as exact numerical values. However, in some real-life situations, the DM may not be able to precisely express the value of the ratings of alternatives with respect to criteria or else he/she uses linguistic expressions. In such situations, when evaluations are based on unquantifiable, incomplete, or unobtainable information, the DM may use other data formats, such as: interval numbers (Jahanshahloo et al., 2006a; Yue, 2011), fuzzy numbers (Chen, 2000; Jahanshaloo et al., 2006b), ordered fuzzy numbers (Roszkowska and Kacprzak, 2016; Kacprzak, 2019), hesitant fuzzy sets (Senvar et al., 2016), intuitionistic fuzzy sets (Boran, Genc et al., 2009) and other.

On the other hand, the increasing complexity of decision problems analysed makes it less feasible to consider all the relevant aspects of the problems by a single DM. Therefore, many real-life problems are considered by a group of DMs. In such situations, the individual decisions made by each DM (usually in the form of an individual decision matrix) are often aggregated to form a collective decision (also in the form of a collective decision matrix). This collective decision is the starting point for the ranking of the alternatives or the selection of the best one.

One of the most popular and often used methods of aggregation, in MCDM methods such as TOPSIS, is arithmetic mean (Chen, 2000; Wang and Chang, 2007; Roszkowska and Kacprzak, 2016). This type of aggregation of individual decisions is also used in practice, e.g., in certain sports, such as snowboard slopestyle or halfpipe. Each participant is evaluated by a group of referees (as DMs) and the average of the referees' scores is taken as the final result for each participant. On the other hand, due to this method of aggregation of individual information, some significant information of the individual decisions of DMs is not taken into consideration. As an example, consider a group of two decision makers who make assessments using the following point scale: $\{1, 2, 3, 4, 5\}$. Let us note that regardless whether their assessment of an alternative with respect to a criterion is in the form "1 and 5", "2 and 4" or "3 and 3", the aggregation results are the same and equal to "3". This means that such an averaged result does not reflect the discrepancies of the individual decisions (preferences of DMs) and that using such averaged information may lead to an incorrect final decision.

The aim of this paper is to present a new approach for ranking of alternatives with fuzzy data for group decision making using the TOPSIS method. In the proposed approach, all individual decision information of DMs is taken into account in determining the ranking of alternatives and selecting the best one. The key stage of this method is the transformation of the decision matrices provided by the decision makers into matrices of alternatives. A matrix corresponding to an alternative is composed of its assessments with respect to all criteria, performed by all the decision makers. Since all individual decision matrices are normalized beforehand with respect to the type of criterion, the positive ideal solution in this approach is a matrix composed of maximal assessments, and the negative ideal solution is a matrix composed of minimal assessments. The distances of alternatives from the PIS and the NIS, in contrast to the classic TOPSIS and to the method based on the aggregation of the individual decisions made by each DM, are the distances between matrices. Using the coefficient of relative closeness of each alternative to the positive ideal solution, a ranking of alternatives is created and the best one is indicated.

The rest of the paper is organized as follows. In Section 2 basic definitions and notations of fuzzy numbers are introduced. In Section 3 the TOPSIS method and its fuzzy extension are presented. The proposed approach and a numerical example are described in Section 4 and Section 5, respectively. Section 6 is devoted to the comparison of the proposed approach with other, similar approaches. Finally, concluding remarks are in Section 7.

2 Fuzzy numbers

In this section some definitions related to fuzzy sets and fuzzy numbers used in the paper are briefly outlined.

Definition 1. (Zadeh, 1965). Let X be a universe set. A fuzzy subset A in a universe of discourse X is characterized by a membership function $\mu_A(x)$ which associates with each element x in X a real number from the interval [0,1]. The function $\mu_A(x)$ is called the grade of membership of x in A.

Definition 2. (Dubois and Prade, 1980). The support of a fuzzy set A is the ordinary subset of X $\text{supp}A = \{x \in X : \mu_A(x) > 0\}$.

Definition 3. (Dubois and Prade, 1980). A fuzzy set A is normalized iff $\exists x \in X, \mu_A(x) = 1$.

Definition 4. (Dubois and Prade, 1980; Zimmermann, 2001). A fuzzy set A is convex iff $\forall\, x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$ $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$.

We can now define the concept of a fuzzy number.

Definition 5. (Dubois and Prade, 1980; Zimmermann, 2001). A fuzzy number A is a convex, normalized fuzzy subset A of the real line $\mathbb{R}$ such that:

a) there exists exactly one $x_0 \in \mathbb{R}, \mu_A(x_0) = 1$ (x_0 is called the mean value of A),
b) $\mu_A(x)$ is piecewise continuous.

If fuzzy subset A of the real line $\mathbb{R}$ is convex and normalized, its membership function is piecewise continuous, and there exists more than one element $x_0 \in \mathbb{R}$, $\mu_A(x_0) = 1$ then A is called a flat fuzzy number (Dubois and Prade, 1980).

In many practical applications of fuzzy numbers, positive triangular and positive trapezoidal fuzzy numbers are used. Figure 1 shows the characteristic points of such numbers, which describe them uniquely. The positive triangular fuzzy number A is denoted by

$$A = (a_A, b_A, c_A), \tag{1}$$

where $0 \le a_A \le b_A \le c_A$, and its membership function is of the form (Fig. 1a)

$$\mu_A(x) = \begin{cases} \frac{x-a_A}{b_A-a_A} & \text{for} \quad a_A \le x \le b_A \\ \frac{c_A-x}{c_A-b_A} & \text{for} \quad b_A \le x \le c_A \end{cases}, \tag{2}$$

while the positive trapezoidal fuzzy number A is denoted by

$$A = (a_A, b_A, c_A, d_A), \tag{3}$$

where $0 \le a_A \le b_A \le c_A \le d_A$, and its membership function is of the form (Fig. 1b)

$$\mu_A(x) = \begin{cases} \frac{x-a_A}{b_A-a_A} & \text{for} \quad a_A \le x \le b_A \\ 1 & \text{for} \quad b_A \le x \le c_A \\ \frac{d_A-x}{d_A-c_A} & \text{for} \quad c_A \le x \le d_A \end{cases}. \tag{4}$$

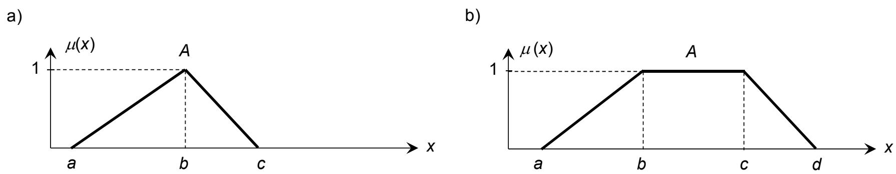

Figure 1: a) A triangular positive fuzzy number A; b) A trapezoidal positive fuzzy number A

If in a positive trapezoid fuzzy number $A = (a_A, b_A, c_A, d_A)$ we have $b_A = c_A$, then A becomes a positive triangular fuzzy number. Let $A = (a_A, b_A, c_A, d_A)$ and $B = (a_B, b_B, c_B, d_B)$ be two positive trapezoidal fuzzy numbers and let $r \in \mathbb{R}$.

Definition 6. The arithmetic operations (used later in the paper) are defined as follows

$$A + B = (a_A + a_B, b_A + b_B, c_A + c_B, d_A + d_B), \tag{5}$$

$$A \cdot B = (a_A \cdot a_B, b_A \cdot b_B, c_A \cdot c_B, d_A \cdot d_B), \tag{6}$$

$$r \cdot A = (r \cdot a_A, r \cdot b_A, r \cdot c_A, r \cdot d_A). \tag{7}$$

In some fuzzy MCDM methods, including fuzzy TOPSIS, it is necessary to measure the distance between fuzzy numbers, and to perform maximum and minimum operations on them.

Definition 7. The distance (d) calculated by the vertex method and the maximum (max) and minimum (min) operations are defined as

$$d(A,B) = \sqrt{\frac{1}{4}[(a_A - a_B)^2 + (b_A - b_B)^2 + (c_A - c_B)^2 + (d_A - d_B)^2]}, \tag{8}$$

$$\max(A,B) = (\max\{a_A, a_B\}, \max\{b_A, b_B\}, \max\{c_A, c_B\}, \max\{d_A, d_B\}), \tag{9}$$

$$\min(A,B) = (\min\{a_A, a_B\}, \min\{b_A, b_B\}, \min\{c_A, c_B\}, \min\{d_A, d_B\}). \tag{10}$$

3 The TOPSIS method

In this section the classical TOPSIS method and its fuzzy extension are presented. Let us assume that the decision maker has to choose one of m possible alternatives described by n criteria. The rating of alternative A_i $(i = 1,...,m)$ with respect to criterion C_j $(j = 1,...,n)$ is denoted by x_{ij}. The set of criteria is divided into two subsets: benefit criteria (greater value is better) denoted by B and cost criteria (lower value is better) denoted by C. Let $W = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights.

The original TOPSIS method assumes that the rating x_{ij} of the alternatives with respect to the criteria, as well as the criteria weights w_j, are expressed precisely by real numbers. It consists of the following steps:

Step 1

Determination of the decision matrix X

$$X = (x_{ij}) \tag{11}$$

where $x_{ij} \in \mathbb{R}$.

Step 2

Calculation of the normalized decision matrix R using vector normalization

$$R = (r_{ij}) \tag{12}$$

where $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}$.

Step 3

Calculation of the weighted normalized matrix V by multiplying the columns of the normalized decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying $\sum_{j=1}^{n} w_j = 1$

$$V = (v_{ij}) \tag{13}$$

where $v_{ij} = r_{ij} \cdot w_j$.

Step 4

Determination of the positive ideal solution A^+

$$A^+ = (v_1^+, v_2^+, \dots, v_n^+) \tag{14}$$

where $v_j^+ = \{(\max_i v_{ij} \text{ if } j \in B), (\min_i v_{ij} \text{ if } j \in C)\}$

and the negative ideal solution A^-

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \tag{15}$$

where $v_j^- = \{(\min_i v_{ij} \text{ if } j \in B), (\max_i v_{ij} \text{ if } j \in C)\}$.

Step 5

Calculation of the Euclidean distances of each alternative A_i from the positive ideal solution A^+

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2} \tag{16}$$

and from the negative ideal solution A^-

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}. \tag{17}$$

Step 6

Calculation of the relative closeness of each alternative A_i to the positive ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{18}$$

Step 7

Ranking of the alternatives A_i according to their relative closeness to the ideal solutions A^+ (the larger the value of RC_i the better the alternative A_i). The best alternative is the one with the largest value of RC_i.

In real-life decision making problems it is usually difficult to express evaluations precisely using real numbers, due to a lack of knowledge and data or to subjective and imprecise expert judgments. In such situations, instead of exact numbers, fuzzy numbers can be used. The fuzzy TOPSIS method based on positive triangular fuzzy numbers proposed by Chen (2000) consists of the following steps:

Step 1

Define the fuzzy decision matrix X

$$X = (x_{ij}) \tag{19}$$

where $x_{ij} = (a_{x_{ij}}, b_{x_{ij}}, c_{x_{ij}})$ is a positive triangular fuzzy number.

Given a group of K decision makers, the rating of alternatives with respect to each criterion can be calculated as $x_{ij} = \frac{1}{K}\left(x_{ij}^1 + x_{ij}^2 + \cdots + x_{ij}^K\right)$, where x_{ij}^k $(k = 1,2,\ldots,K)$ is the rating of alternative i with respect to criterion j provided by decision maker k.

Step 2

Construct the normalized fuzzy decision matrix R using linear normalization

$$R = \left(r_{ij}\right) \tag{20}$$

where

$$r_{ij} = \begin{cases} \left(\frac{a_{x_{ij}}}{\max_i c_{x_{ij}}}, \frac{b_{x_{ij}}}{\max_i c_{x_{ij}}}, \frac{c_{x_{ij}}}{\max_i c_{x_{ij}}}\right) & \text{if} \quad j \in B \\ \left(\frac{\min_i a_{x_{ij}}}{c_{x_{ij}}}, \frac{\min_i a_{x_{ij}}}{b_{x_{ij}}}, \frac{\min_i a_{x_{ij}}}{a_{x_{ij}}}\right) & \text{if} \quad j \in C \end{cases} . \tag{21}$$

Step 3

Construct the weighted normalized fuzzy matrix V by multiplying the columns of the normalized fuzzy decision matrix R by the associated weights $w_j \in \mathbb{R}$ satisfying $\sum_{j=1}^{n} w_j = 1$

$$V = \left(v_{ij}\right) \tag{22}$$

where $v_{ij} = r_{ij} \cdot w_j = \left(a_{r_{ij}}, b_{r_{ij}}, c_{r_{ij}}\right) \cdot w_j = \left(a_{r_{ij}} \cdot w_j, b_{r_{ij}} \cdot w_j, c_{r_{ij}} \cdot w_j\right)$.

Step 4

Determine the fuzzy positive ideal solution as follows

$$A^+ = (v_1^+, v_2^+, \ldots, v_n^+) \tag{23}$$

where $v_j^+ = \max_i v_{ij}$ and the fuzzy negative ideal solution

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-) \tag{24}$$

where $v_j^- = \min_i v_{ij}$.

Step 5

Calculate the distances of each alternative A_i from the positive ideal solution A^+

$$d_i^+ = \sum_{j=1}^{n} dd\left(v_{ij}, v_j^+\right) \tag{25}$$

and from the negative ideal solution A^-

$$d_i^- = \sum_{j=1}^{n} dd\left(v_{ij}, v_j^-\right) \tag{26}$$

where the distance dd between two positive triangular fuzzy numbers $A = (a_A, b_A, c_A)$ and $B = (a_B, b_B, c_B)$ is equal to

$$dd(A, B) = \sqrt{\frac{1}{3}\left[(a_A - a_B)^2 + (b_A - b_B)^2 + (c_A - c_B)^2\right]}. \tag{27}$$

Step 6

Calculate the relative closeness of alternative A_i to the ideal solution A^+

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-} . \tag{28}$$

Step *7*

Rank the alternatives A_i and select the one with the largest value of RC_i.

4 The proposed approach

In this section the proposed approach is presented. Consider an MCDM problem for group decision making. Let $\{A_1, A_2, \dots, A_m\}$ $(m \geq 2)$ be a discrete set of m feasible alternatives, $\{C_1, C_2, \dots, C_n\}$ $(n \geq 2)$ be a finite set of criteria, $w = (w_1, w_2, \dots, w_n)$ be the vector of criteria weights, such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. Let $\{DM_1, DM_2, \dots, DM_K\}$ $(K \geq 2)$ be a group of decision makers.

In the process of group decision making, the DMs are asked to assess alternatives with respect to criteria. In many real-life situations, when the DMs' knowledge of the analysed subject is incomplete, or the available data are inaccurate, or when the ratings are expressed linguistically, fuzzy numbers can be used. In that case, each DM provides a decision matrix of the form

$$X^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{bmatrix} \end{array} \tag{29}$$

where $x_{ij}^k = \left(a_{x_{ij}^k}, b_{x_{ij}^k}, c_{x_{ij}^k}, d_{x_{ij}^k}\right)$ is a positive trapezoidal fuzzy number representing the rating of alternative A_i $(i = 1,2, \dots, m)$ with respect to criterion C_j $(j = 1,2, \dots, n)$ provided by decision maker DM_k $(k = 1,2, \dots, K)$.

The decision matrix X^k (29) can be constructed in various ways, for instance, using crisp evaluations x_{ij}^{k*}. The transformation is carried out by extending the support and kernel of the crisp evaluation to the estimated or assumed imprecision bound of evaluation. For empirical data from the range $[L, U]$, the crisp value $x_{ij}^{k*} \in [L, U]$ can be transformed into the trapezoidal fuzzy number $\left(a_{x_{ij}}, b_{x_{ij}}, c_{x_{ij}}, d_{x_{ij}}\right)$, where $a_{x_{ij}} = \max\{L, x_{ij}^{k*} - 2\sigma\}$, $b_{x_{ij}} = \max\{L, x_{ij}^{k*} - \sigma\}$, $c_{x_{ij}} = \min\{U, x_{ij}^{k*} + \sigma\}$, $d_{x_{ij}} = \min\{U, x_{ij}^{k*} + 2\sigma\}$ where σ is the assumed or estimated imprecision bound of empirical data (for more details and a numerical example, see Rudnik and Kacprzak, 2017). Another very popular way of constructing the fuzzy decision matrix X^k (29) is to use linguistic variables to evaluate the ratings of alternatives with respect to various criteria (for more details, see e.g. Bonissone and Decker,1986; Shemshadi et al., 2011; Kacprzak, 2017; Hatami-Marbini and Kangi, 2017).

Next, in order to ensure comparability of criteria, the fuzzy decision matrix X^k is normalized. The normalized fuzzy decision matrix

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ Y^k = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} y_{11}^k & y_{12}^k & \cdots & y_{1n}^k \\ y_{21}^k & y_{22}^k & \cdots & y_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1}^k & y_{m2}^k & \cdots & y_{mn}^k \end{bmatrix} \end{array} \tag{30}$$

is calculated using the following formulas

$$y_{ij}^k = \begin{cases} \left(\frac{a_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{b_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{c_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}, \frac{d_{x_{ij}^k}}{\max_i d_{x_{ij}^k}}\right) & \text{if} \quad j \in B \\ \left(\frac{\min_i a_{x_{ij}^k}}{d_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{c_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{b_{x_{ij}^k}}, \frac{\min_i a_{x_{ij}^k}}{a_{x_{ij}^k}}\right) & \text{if} \quad j \in C \end{cases}. \tag{31}$$

Using the vector of criteria weights $w = (w_1, w_2, \dots, w_n)$, the weighted normalized fuzzy decision matrix is calculated for each DM

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ V^k = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} v_{11}^k & v_{12}^k & \cdots & v_{1n}^k \\ v_{21}^k & v_{22}^k & \cdots & v_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1}^k & v_{m2}^k & \cdots & v_{mn}^k \end{bmatrix} \end{array} \tag{32}$$

where $v_{ij}^k = w_j y_{ij}^k = \left(w_j a_{y_{ij}^k}, w_j b_{y_{ij}^k}, w_j c_{y_{ij}^k}, w_j d_{y_{ij}^k}\right)$. The matrices V^k form the basis for the construction of weighted normalized fuzzy decision matrices for each alternative A_i

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ W^i = \begin{array}{c} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{array} & \begin{bmatrix} v_{i1}^1 & v_{i2}^1 & \cdots & v_{in}^1 \\ v_{i1}^2 & v_{i2}^2 & \cdots & v_{in}^2 \\ \vdots & \vdots & \ddots & \vdots \\ v_{i1}^K & v_{i2}^K & \cdots & v_{in}^K \end{bmatrix} \end{array}. \tag{33}$$

Matrices W^i constitute the basis for the construction of the ranking of the alternatives and the selection of the best one using the fuzzy TOPSIS method. The positive ideal solution A^+ is determined as follows

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ A^+ = \begin{array}{c} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{array} & \begin{bmatrix} v_1^{1+} & v_2^{1+} & \cdots & v_k^{1+} \\ v_1^{2+} & v_2^{2+} & \cdots & v_n^{2+} \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{K+} & v_2^{K+} & \cdots & v_n^{K+} \end{bmatrix} \end{array} \tag{34}$$

where $v_j^{k+} = \max_i v_{ij}^k$, and the negative ideal solution A^- is determined as follows

$$\begin{array}{cc} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ A^- = \begin{array}{c} DM_1 \\ DM_2 \\ \vdots \\ DM_K \end{array} & \begin{bmatrix} v_1^{1-} & v_2^{1-} & \cdots & v_k^{1-} \\ v_1^{2-} & v_2^{2-} & \cdots & v_n^{2-} \\ \vdots & \vdots & \ddots & \vdots \\ v_1^{K-} & v_2^{K-} & \cdots & v_n^{K-} \end{bmatrix} \end{array} \tag{35}$$

where $v_j^{k-} = \min_i v_{ij}^k$. Next, the distances of each alternative A_i represented by matrix W^i from PIS

$$d_i^+ = \sum_{k=1}^K \sum_{j=1}^n d\left(v_{ij}^k, v_j^{k+}\right) \tag{36}$$

and from NIS

$$d_i^- = \sum_{k=1}^K \sum_{j=1}^n d\left(v_{ij}^k, v_j^{k-}\right) \tag{37}$$

are calculated. Using these distances, the relative closeness coefficients RC_i to PIS for each alternative A_i is calculated

$$RC_i = \frac{d_i^-}{d_i^- + d_i^+}. \tag{38}$$

According to the descending values of RC_i, all alternatives A_i are rank ordered and the best one is selected.

Remark 1

Note that if in the proposed approach we use triangular fuzzy numbers and the distance measure between two triangular fuzzy numbers (27), and if there is only one DM, i.e. $K = 1$, then the proposed approach is equivalent to the fuzzy TOPSIS method proposed by Chen (2000).

Remark 2

Note that if we take into account the form of the matrices W^i, the positive ideal solution A^+ and the negative ideal solution A^-, the proposed approach can be regarded as a simultaneous application of the fuzzy TOPSIS for each of the DMs represented by the corresponding rows of these matrices.

5 Numerical example

In this section our new approach is presented on a numerical example. Consider a fuzzy MCDM problem for group decision making, consisting of the set of three feasible alternatives $\{A_1, A_2, A_3\}$ rated with respect to the set of three benefit criteria $\{C_1, C_2, C_3\}$ by a group of three decision makers $\{DM_1, DM_2, DM_3\}$, with the vector of criteria weights $w = (0.4, 0.2, 0.4)$. The DMs have used trapezoidal fuzzy numbers to rate the alternatives with respect to the criteria and their evaluations are shown in Table 1. Using formula (31), the decision matrices are normalized and using the vector w of criteria weights, the weighted normalized fuzzy decision matrix is calculated for each DM (see Table 2). Next, these matrices are transformed into the weighted normalized fuzzy decision matrices for each alternative (see Table 3). Using these matrices, the positive ideal solution A^+ and the negative ideal solution A^- are determined (see Table 4). Finally, the distances of each alternative from the positive ideal solution d_i^+ and from the negative ideal solution d_i^+ are calculated (see Table 5). This allows to calculate the relative closeness coefficient RC_i and the rank order R of the alternatives (where $\prec$ means "inferior to"):

$$A_3 \prec A_2 \prec A_1.$$

Hence, alternative A_1 should be selected.

Table 1: Individual decision matrices provided by the decision makers

		C_1	C_2	C_3
DM_1	A_1	(72,78,84,90)	(72,77,82,87)	(85,87,89,91)
	A_2	(77,78,79,80)	(69,77,85,93)	(83,85,87,89)
	A_3	(80,85,90,95)	(59,68,77,86)	(80,82,84,86)
DM_2	A_1	(77,79,81,83)	(68,73,78,83)	(82,85,88,91)
	A_2	(93,95,97,99)	(76,79,82,85)	(65,72,79,86)
	A_3	(79,81,83,85)	(72,79,86,93)	(81,84,87,90)
DM_3	A_1	(85,86,87,88)	(76,79,82,85)	(80,86,92,98)
	A_2	(79,82,85,88)	(81,84,87,90)	(81,85,89,93)
	A_3	(77,80,83,86)	(84,86,88,90)	(81,84,87,90)

Table 2: Weighted normalised decision matrices

		C_1	C_2	C_3
DM_1	A_1	(0.3032,0.3284,0.3537,0.3789)	(0.1548,0.1656,0.1763,0.1871)	(0.3736,0.3824,0.3912,0.4000)
	A_2	(0.3242,0.3284,0.3326,0.3368)	(0.1484,0.1656,0.1828,0.2000)	(0.3648,0.3736,0.3824,0.3912)
	A_3	(0.3368,0.3579,0.3789,0.4000)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
DM_2	A_1	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.3604,0.3736,0.3868,0.4000)
	A_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1763,0.1828)	(0.2857,0.3165,0.3473,0.3780)
	A_3	(0.3192,0.3273,0.3354,0.3434)	(0.1548,0.1699,0.1849,0.2000)	(0.3560,0.3692,0.3824,0.3956)
DM_3	A_1	(0.3864,0.3909,0.3955,0.4000)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3510,0.3755,0.4000)
	A_2	(0.3591,0.3727,0.3864,0.4000)	(0.1800,0.1867,0.1933,0.2000)	(0.3306,0.3469,0.3633,0.3796)
	A_3	(0.3500,0.3636,0.3773,0.3909)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3429,0.3551,0.3673)

Table 3: Weighted normalised decision matrices for the alternatives

		C_1	C_2	C_3
A_1	DM_1	(0.3032,0.3284,0.3537,0.3789)	(0.1548,0.1656,0.1763,0.1871)	(0.3736,0.3824,0.3912,0.4000)
	DM_2	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.3604,0.3736,0.3868,0.4000)
	DM_3	(0.3864,0.3909,0.3955,0.4000)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3510,0.3755,0.4000)
A_2	DM_1	(0.3242,0.3284,0.3326,0.3368)	(0.1484,0.1656,0.1828,0.2000)	(0.3648,0.3736,0.3824,0.3912)
	DM_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1763,0.1828)	(0.2857,0.3165,0.3473,0.3780)
	DM_3	(0.3591,0.3727,0.3864,0.4000)	(0.1800,0.1867,0.1933,0.2000)	(0.3306,0.3469,0.3633,0.3796)
A_3	DM_1	(0.3368,0.3579,0.3789,0.4000)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
	DM_2	(0.3192,0.3273,0.3354,0.3434)	(0.1548,0.1699,0.1849,0.2000)	(0.3560,0.3692,0.3824,0.3956)
	DM_3	(0.3500,0.3636,0.3773,0.3909)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3429,0.3551,0.3673)

Table 4: Positive ideal solution and negative ideal solution

		C_1	C_2	C_3
A^+	DM_1	(0.3368,0.3579,0.3789,0.4000)	(0.1548,0.1656,0.1828,0.2000)	(0.3736,0.3824,0.3912,0.4000)
	DM_2	(0.3758,0.3838,0.3919,0.4000)	(0.1634,0.1699,0.1849,0.2000)	(0.3604,0.3736,0.3868,0.4000)
	DM_3	(0.3864,0.3909,0.3955,0.4000)	(0.1867,0.1911,0.1956,0.2000)	(0.3306,0.3510,0.3755,0.4000)
A^-	DM_1	(0.3032,0.3284,0.3326,0.3368)	(0.1269,0.1462,0.1656,0.1849)	(0.3516,0.3604,0.3692,0.3780)
	DM_2	(0.3111,0.3192,0.3273,0.3354)	(0.1462,0.1570,0.1677,0.1785)	(0.2857,0.3165,0.3473,0.3780)
	DM_3	(0.3500,0.3636,0.3773,0.3909)	(0.1689,0.1756,0.1822,0.1889)	(0.3265,0.3429,0.3551,0.3673)

Table 5: The distances of each alternative from the positive ideal solution d_i^+, the negative ideal solution d_i^-, the relative closeness coefficients RC_i and the ranking order R of alternatives

	d_i^+	d_i^-	RC_i	R
A_1	0.1338	0.1602	0.5448	**1**
A_2	0.1494	0.1467	0.4954	**2**
A_3	0.1523	0.1338	0.4677	**3**

6 Comparison of the proposed approach with other and similar approaches

In this section, the proposed approach is compared with other similar methods. Figures 2, 3a and 3b show the hierarchical structure of the classical TOPSIS (Hwang and Yoon, 1981), the TOPSIS for group decision making with aggregation of individual decision matrices, and the proposed approach, respectively.

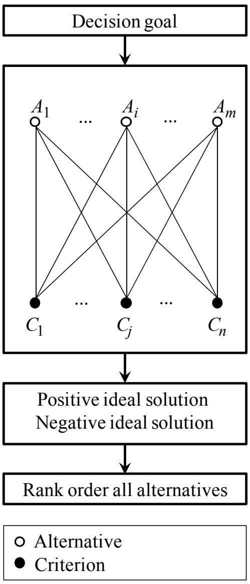

Figure 2: The hierarchical structure of the classical TOPSIS

Here the results (rankings of alternatives) using the proposed method ($\boldsymbol{PA}$) are compared with the method which aggregates the individual weighted normalized decision matrices into an aggregated collective matrix (which in TOPSIS is the starting point for the ranking of alternatives), based on data from the example in Section 5 (Table 1). For the calculation of the aggregated collective matrix $X = \left(x_{ij}\right)$ the following aggregation methods, known from the literature, are used:

- $\boldsymbol{AGG1}$ – arithmetic mean (Chen, 2000; Wang and Chang, 2007; Roszkowska and Kacprzak, 2016), defined by

$$x_{ij} = \frac{1}{K}\sum_{k=1}^{K} x_{ij}^{k} = \left(\frac{1}{K}\sum_{k=1}^{K} a_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} b_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} c_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} d_{x_{ij}^{k}}\right),$$

- $\boldsymbol{AGG2}$ – geometric mean (Shih et al., 2007; Ye and Li, 2009), defined by

$$x_{ij} = \left(\prod_{k=1}^{K} x_{ij}^{k}\right)^{\frac{1}{K}} =$$
$$\left(\left(\prod_{k=1}^{K} a_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} b_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} c_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} d_{x_{ij}^{k}}\right)^{\frac{1}{K}}\right),$$

- $\boldsymbol{AGG3}$ – modified arithmetic mean (Shemshadi et al., 2011; Nadaban et al., 2016), defined by

$$x_{ij} = \left(\min_k a_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} b_{x_{ij}^{k}}, \frac{1}{K}\sum_{k=1}^{K} c_{x_{ij}^{k}}, \max_{\mathrm{k}} d_{x_{ij}^{k}}\right),$$

- $\boldsymbol{AGG4}$ – modified geometric mean (Ding, 2011; Chang et al., 2009; Hatami--Marbini and Kangi, 2017), defined by

$$x_{ij} = \left(\min_k a_{x_{ij}^{k}}, \left(\prod_{k=1}^{K} b_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \left(\prod_{k=1}^{K} c_{x_{ij}^{k}}\right)^{\frac{1}{K}}, \max_k d_{x_{ij}^{k}}\right).$$

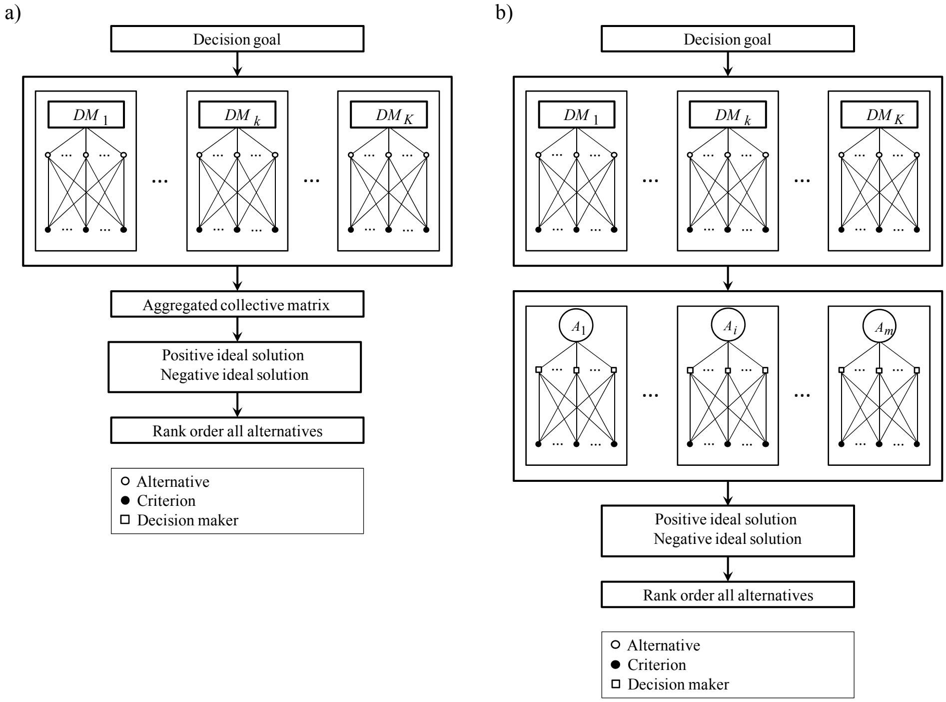

Figure 3: a) The hierarchical structure of the TOPSIS method for group decision making with aggregation of individual decision matrices, b) The hierarchical structure of the proposed TOPSIS method for group decision making

Table 6 shows the distance of each alternative A_i from the positive ideal solution d_i^+ and the negative ideal solution d_i^-, as well as the relative closeness coefficients RC_i and rank order R of the alternatives using the proposed method and different aggregation methods. The last column, denoted by J, consists of the normalized (summing up to 1) values of the relative closeness coefficients of each alternative to the ideal solution, which allows to highlight the differences between the final scores of the alternatives. Next, Table 7 and Fig. 4 show the ranking of the alternatives. Let us note that the aggregation methods using arithmetic mean and geometric mean give the same rank order of the alternatives: $A_3 \prec A_1 \prec A_2$, but different from that of the proposed approach. These methods swap the order of alternatives A_1 and A_2. This means that the final ranking order of the alternatives and the choice of the best one depend on the method used. Let us also note that the modified arithmetic mean and the modified geometric mean result in the same rank order of the alternatives $A_2 \prec A_3 \prec A_1$, which is also different from the proposed approach and from the aggregation methods using arithmetic mean and geometric mean. In these cases, alternative A_1 is the best; the results are the same as those obtained using the proposed method. Taking into account column J in Table 6 and Figure 4b, we can notice that the aggregation methods using arithmetic mean and geometric mean result in a fairly high score of alternative A_2 and a fairly low final score of alternative A_3 in comparison with the other methods analysed (which result in less diverse final scores).

Table 6: The results obtained using the proposed method and different aggregation methods

	ALT.	d_i^+	d_i^-	RC_i	R	J
PA	A_1	0.1338	0.1602	0.5448	1	0.3613
	A_2	0.1494	0.1467	0.4954	2	0.3285
	A_3	0.1523	0.1338	0.4677	3	0.3102
$AGG1$	A_1	0.0253	0.0226	0.4721	2	0.3280
	A_2	0.0220	0.0267	0.5478	1	0.3806
	A_3	0.0318	0.0230	0.4194	3	0.2914
$AGG2$	A_1	0.0244	0.0242	0.4980	2	0.3405
	A_2	0.0230	0.0261	0.5322	1	0.3639
	A_3	0.0315	0.0240	0.4322	3	0.2955
$AGG3$	A_1	0.0319	0.0473	0.5971	1	0.3484
	A_2	0.0354	0.0431	0.5492	3	0.3204
	A_3	0.0405	0.0532	0.5677	2	0.3312
$AGG4$	A_1	0.0316	0.0475	0.6004	1	0.3498
	A_2	0.0356	0.0428	0.5460	3	0.3181
	A_3	0.0401	0.0532	0.5699	2	0.3320

Table 7: The rankings of alternatives based on the relative closeness coefficients

	RANKING - RC_i
PA	$A_3 \prec A_2 \prec A_1$
$AGG1$	$A_3 \prec A_1 \prec A_2$
$AGG2$	$A_3 \prec A_1 \prec A_2$
$AGG3$	$A_2 \prec A_3 \prec A_1$
$AGG4$	$A_2 \prec A_3 \prec A_1$

a) b)

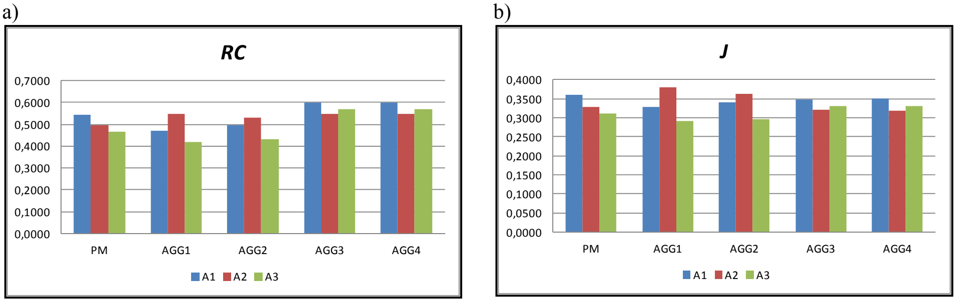

Figure 4: The rankings of alternatives based on: a) the relative closeness coefficients (RC_i), b) the normalized relative closeness coefficients (J)

7 Conclusions

In this paper an extended TOPSIS method based on fuzzy numbers for group decision making problems has been presented. Most papers in the literature aggregate the individual decision matrices provided by the DMs into a collective decision matrix which is the starting point for the ranking of alternatives or the selection of the best one, using arithmetic mean, geometric mean or their modifications. Such an averaged result does not reflect the discrepancies between the individual assessments or the preferences of the DMs. By contrast, in the proposed approach, all individual decision data of the DMs are taken into account in determining the ranking of alternatives and the selection of the best one.

The numerical example has shown that the proposed approach, as compared with other methods of aggregation of individual decision matrices of each DMs, can give a different final result, both as regards the ranking of alternatives and the selection of the best one.

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