



Krzysztof Borowski

 <https://orcid.org/0000-0003-0434-7573>

Institute of Risk and Financial Market
Warsaw School of Economics, Warsaw, Poland
kborow@sgh.waw.pl

Testing 65 equity indexes for normal distribution of returns

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Abstract

Aim/purpose – The primary aim of the paper is to verify the hypothesis on the normal distributions of 65 stock index returns, while the secondary aims are to examine normal distributions for specific years (for six indexes) and for bull and bear markets (for DJIA), demonstrate that the distribution of rates of return for individual indexes can be normal in short time intervals, and then rank analyzed indexes according to the proximity of the distribution of their rates of return to the normal distribution.

Design/methodology/approach – The research sample consists of the value of 65 stock indexes from various time intervals. The sample includes both developed markets and emerging markets. The following rates of return were tested for the normality of the rate of return distribution: close-close, open-open, open-close and overnight, which were calculated for daily, weekly, monthly, quarterly and yearly data. Statistical tests of different properties and forces were used: Jarque–Bera (JB), Lilliefors (L), Cramer von Mises (CVM), Watson (W), Anderson–Darling (AD). In the case of six indexes of developed markets (DJIA, SP500, DAX, CAC40, FTSE250 and NIKKEI225), normality tests of rates distribution were calculated for individual years 2013-2016 (daily data). In case of the DJIA index, the normality tests of the distribution of returns for individual bull and bear markets were analyzed (daily data, rates of return close-close). In the last part of the paper the analyzed indexes were ranked due to the convergence of their return to normal distribution with the use of the following tests: Jarque–Bera, Shapiro–Wilk and D’Agostino-Pearson.

Findings – The distribution of daily and weekly returns of equity indexes is not a normal distribution for all analyzed rates of return. For quarterly and annual data compression the smallest number when there were no reasons to reject the null hypothesis was observed for overnight returns compared to close-close, open-close and open-open returns. For the daily, weekly and monthly overnight rates of return, the null hypothesis was

rejected for all analyzed indexes. The following general conclusion can be formulated: the higher the data compression (from daily to yearly), the fewer rejections of H_0 hypothesis. The distribution of daily returns can be normal only in given (rather short) time intervals, e.g., particular years or up or down waves (bull and bear markets). The position of the index in the ranking is not dependent on the date of its first publication, and hence on the number of rates of return possible to calculate for analyzed index, but only on the distribution of its rates of return.

Research implications/limitations – The main limitations of the obtained results are different time horizons of each of the analyzed indexes (from the first date in a data base until 30.06.2017). The major part of the returns of the analyzed indexes differs from the normal distribution, which question the possibility of unreflective implementation in practice of economic such models as CAPM and its derivatives, Black–Scholes options valuation, portfolio theory and efficient market hypothesis, especially in long time horizons.

Contribution/value/contribution – The contribution of this paper is verification of the statistical hypothesis regarding normal distribution of rates of return: (1) other than close-close, i.e. open-open, open-close and overnight with the use of various statistical tests, various data compression (daily, weekly, monthly, quarterly, yearly) for 65 indexes, (2) for six stock exchange indexes in each of the years from the period of 2013-2016 (daily data) and (3) for individual up and down waves for the DJIA index (daily data). In addition, other papers focused only on one or two statistical tests, while five different tests were implemented in this paper. This paper is the first to create a ranking of stock market indexes due to the normal distribution.

Keywords: financial markets, distribution of rate of returns, capital market efficiency.

JEL Classification: G10, G14.

1. Introduction

One of the most important assumptions in theoretical and empirical research in finance is that relevant variables (e.g., rates of return) are characterized by normal distribution. For longer time periods, the Central Limit Theorem (Lindeberg, 1922) is often invoked as an argument for the normal distribution. Even if the daily returns are non-normal, the Central Limit Theorem predicates that the sum of N independent, identically distributed random variables with finite variance converges to a normal distribution, when N is large. From a probabilistic point of view, it is not at all obvious that the assumptions of the Central Limit Theorem are satisfied. The assumptions of normal distribution and constant mean and variance are standard in financial analysis. The assumption of normal distribution of the stock returns is incorporated in the most popular and most used models in the theory and practice of financial economics. Among them are: the Markowitz Portfolio Theory (Markowitz, 1952), Capital Asset Pricing Model (Sharpe, 1964), and the Consumption CAPM (Lucas, 1978). Ad-

ditionally, the Black–Scholes option pricing model (Black & Scholes, 1973; Merton, 1973) is based on the assumption that equity prices follow a geometric Brownian motion process, which has normally distributed increments. Another requirement to fulfill the assumption of the normal distribution is that equity markets are rational and efficient. According to that logic, if return expectations implicit in asset prices are rational, actual rates of return should be normally distributed around these expectations (Bodie, Kane, & Marcus, 2014, p. 135).

In the process of mean-variance efficiency, small sample results have been derived under the assumption of normal distribution of returns (Affleck-Graves & McDonalds, 1989; Bookstaber & McDonalds, 1987; Clark, 1973; Fama, 1965, 1976, pp. 123-156; Harris, 1986; Richardson & Smith, 1993). But empirical evidence strongly rejects normal distribution and shows that stock returns have leptokurtic distribution and skewness (both left and right). Some authors emphasize that violation of the assumption may lead to incorrect inference (MacKinlay & Richardson, 1991). The general conclusion of research claims that equity returns are not normally distributed, thus questioning all obtained results relying on the assumption of normally distributed returns. Hence, it is important to know if rates of return are in fact normally distributed.

The primary aim of this paper is to verify an existing hypothesis that stock index returns are normally distributed. This paper examines 65 stock indexes for different interest rates (close-close, open-open, open-close, and overnight). The paper also examines: return rates of six indexes over the period of 2013-2016; and, examines the rates of return of DJIA in bull and bear markets. Finally, the paper ranks the analyzed indexes according the proximity of the distribution of the return rates relative to normal distribution.

The paper consist of six sections. The introduction states the aims of the paper while the literature review examines previous findings in the research field of the normal distribution rates of return on financial market. This is followed by a description of the research methodology, the research results, a discussion of findings, and the conclusion. References are provided in the final section.

2. Literature review of the normal distribution of rates of return

One of the earliest works dedicated to the distribution of rates on the financial markets was Bachelier's study (1900), who found that the price differences in subsequent periods were normal distributed variables, using random walk model of financial instrument prices. The expected value of the instrument price

change was zero, and the variance of price change was a function of the length of the analyzed period. The more advanced study of time series was carried out by Kendall (1953), which, on the basis of weekly rates of return from the British market, found not only normal distribution of price changes but also their leptokurtosis. Another important study was published by Osborne (1959), who found the normal distribution of the returns of the companies listed on the American Stock Exchange and the New York Stock Exchange.

Mandelbrot (1963, 1967) and Fama (1965) were the first authors who found that empirical distribution of equity rates of returns were not normal. Mandelbrot (1963, 1967) presented evidence that distributions of returns can be well approximated by the stable Paretian distribution with a characteristic exponent less than 2. The research of Fama (1965), based on a sample of 30 stocks of DJIA index, as well as other papers (Mantegna & Stanley, 1995, 2000, pp.124-176; Mittnik, Rachev, & Paoella, 1998) confirmed Mandelbrot (1963) observations.

Officer (1972) found that monthly returns follow normality, and that the standard deviation appears to be a well behaved measure of scale. Clark (1973) suggested that the lognormal distribution may be a better fit of the data sample of cotton futures prices than a stable Paretian distribution. Praetz (1972), using weekly rates of return for the Sydney Stock exchange shares, concluded that the t (Student) distribution is a better approximation than the stable Paretian. Blattberg & Gonedes (1974) using a daily and weekly data sample of the DJIA took into consideration three distributions: t (Student), normal, and Cauchy. They concluded that the t (Student) was a better representation than the normal distribution for daily returns, but a normal distribution applied better to monthly returns. Fama (1976) rejected the hypothesis that the monthly returns of 14 out of 30 Dow Jones Industrial components were normally distributed in the period of 1951-1968.

Hagerman (1978) rejected the normal distribution and proposed to use an alternative distribution as a mix of the normal and the t (Student) distributions, but Akgiray & Booth (1987) found that normal distribution was a good fit for the monthly stock returns. For describing security returns, Bookstaber & McDonald (1987) introduced the generalized distribution GB2, which represents extremely flexible distribution, containing a large number of well-known distributions, such as the lognormal, log- t , and log-Cauchy distributions, as special or limiting cases and allowing large, even infinitely higher moments. The research of Gray & French (1990), based on the S&P500 index, used three different distributions (scaled- t , logistic, and exponential power) to model log stock index returns.

Aparicio & Estrada (2001), on the basis of daily data of 13 European countries, compared four distributions: logistic, scaled-t, exponential power and a mixture of two normal distributions. They found the scaled-t distribution to be the most appropriate fit for the data sample. Linden (2001) analyzed the distribution of rates of return (daily, weekly and monthly) for the 20 most traded share of the Helsinki Stock Exchange and found that the daily returns were better fitted by asymmetric Laplace than by the normal distribution. Aas (2004), on the base of the rates of return for Norwegian, American, German, and Japanese stock markets in the period of 1970-2002, observed that the fit to the normal distribution for the Norwegian and Japanese market was quite good in the left tail of the distribution, but not good in the right tail.

Malevergne, Pisarenko, & Sornette (2005) analyzed daily data of DJIA and 5-minutes returns of the Nasdaq Composite Index as well as the 1-minute returns of the S&P500. They proposed a parametric representation of the tail of the distribution of returns encompassing both a regularly varying distribution in one limit of the parameters and rapidly varying distribution of the class of the Stretched-Exponential (SE) and the log-Weibull or Stretched Log-Exponential (SLE) distributions in other limits. Rachev, Stoyanov, Biglova, & Fabozzi (2005), analyzing a sample of daily returns for 382 U.S. stocks, found that the stable Paretian hypothesis better explains the tails and the central part of the returns distribution.

Amongst the more contemporary research, special consideration should be given to the work of Scalas & Kim (2007) who using a stable distribution approximated the daily rates of return for the DJIA and MIBTEL indexes. For this first index, the Kolmogorov and chi-square tests confirmed, and for the second index, they denied the hypothesis that index returns could be approximated by a stable distribution. Egan (2007) examined the fit of three different statistical distributions to the returns of the S&P500 Index from 1950-2005, finding that the both normal and lognormal distributions were a poor fit to the daily percentage returns of the analyzed index. In the work of Barunik, Vacha, & Vošvrda (2010), the hypothesis of a normal distribution of returns was rejected for the WIG, PX and BUX indexes from March 2005 through March 2009. Baradaran-Ghahfarokhi & Baradaran-Ghahfarokhi (2009) found the same for the following indexes: CAC40, DAX, DJAC, FTSE100, ISEQ, and S&P500. For the German and the American markets, the fit was relatively good in the right tail, but not so good in the left. Value at Risk (VaR) calculated with the use of the stable distribution are closer to real distribution of these indexes than the above mentioned

index. Chalabi, Scott, & Wuertz (2012) used the generalized lambda distribution (GLD) family as a flexible distribution to model financial data sets. Corlu, Metrelliyoz, & Tiniç (2016) found the generalized lambda distribution (of skewed Student t-distribution, Jonson system of distribution, the normal inverse Gaussian distribution and the g-and-h distribution) to be the most appreciable fit of daily equity index rate of returns for the period of 1979-2014.

Naumoski, Gaber, & Gaber-Naumoska (2017) investigated rates of return for Southeast European emerging countries stock exchanges, and with the use of the Anderson–Darling test, rejected the assumption of normal distribution for all considered data samples and found that the daily stock returns are best fitted by the Johnson SU distribution, whereas for the weekly and monthly stock returns there were many distributions that could be considered a best fit.

Barunik et al. (2010) analyzed the normality of returns distribution from March 2005 to March 2009 for the following equity indexes: PX, WIG, BUX, DAX, and S&P500. The analyzed period was divided into two sub-periods: the first half of the data represented the pre-crisis period and the second half represented post-crisis data. The first period, in comparison to the second period was better described by the normal distribution (except for the PX index). Otherwise, the real data was characterized by larger-than-normal but smaller-than-stable tails.

Bolt & Miłobędzki (1994), analyzing the rates of return for the WIG index and 21 stocks listed on the Warsaw Stock Exchange during the period 1991-1993, concluded that they were not normally distributed. In turn, Fiszeder (2000) conducted a study of WIG index returns and 12 other world indexes during the period January 1997 through June 1999 with the following compliance tests: Pearson, Kolmogorov–Lilliefors and Shapiro–Wilk. The first two tests rejected the null hypothesis regarding the normal returns distribution for all tested indexes. In the case of the Shapiro–Wilk test, there was no reason to reject the null hypothesis except for the NIKKEI225 index. Rokita (2000) calculating rates of return for the WIG20 index in the period of 13.09.1997-15.02.2000, came to the conclusion that it was not normally distributed. These results were confirmed by Osińska (2006, pp. 134-167), who analyzed the rates of return of the indexes WIG20, WIG and the 18 components of the latter, from January 1999 to July 2001. Also Witkowska & Kompa (2007) analyzed returns for 12 companies and two Warsaw Stock Exchange indexes in the period of 1.01.2003-31.12.2005. Those results did not follow a normal distribution.

3. Research methodology

The research area consists of three parts.

In the first part, the hypothesis of the normal distribution of returns of 65 equity indexes was verified. The list of the analyzed indexes and the first date of each index included in the calculation is presented in Table 3 (Appendix) – the data was obtained from Reuters. The statistical hypothesis for each of analyzed indexes was verified for the following time intervals: daily, weekly, monthly, quarterly and yearly.

For each of the analyzed indexes the following rates of return were calculated (daily rates of return):

- a) Close-Close (C-C): $\ln\left(\frac{C_t}{C_{t-1}}\right)$ (last session close vs previous session close),
- b) Overnight (OV): $\ln\left(\frac{O_t}{C_{t-1}}\right)$ (last session open vs previous session close),
- c) Open-Open (O-O): $\ln\left(\frac{O_t}{O_{t-1}}\right)$ (last session open vs previous session open),
- d) Open-Close (O-C): $\ln\left(\frac{C_t}{O_t}\right)$ (last session close vs last session open),

where:

C_t – closing price in the period t ,

C_{t-1} – closing price in the period $t-1$,

O_t – open price in the period t ,

O_{t-1} – open price in the period $t-1$.

The choice of the above rates of return results from two premises. The first is the investment one – a transaction takes place at strictly defined moments of the session at the opening or closing prices. The other derives of earlier scientific papers, most research concentrates solely on the close-close rates.

In the case of arithmetic returns, it is easy to prove relations between them:

$${}_{t \rightarrow t+1}^{C-C}r = {}_{t \rightarrow t+1}^{OV}r + {}_{t+1}^{O-C}r + {}_{t \rightarrow t+1}^{OV}r \cdot {}_{t+1}^{O-C}r$$

$${}_{t \rightarrow t+1}^{O-O}r = {}_{t \rightarrow t+1}^{OV}r + {}_t^{O-C}r + {}_{t \rightarrow t+1}^{OV}r \cdot {}_t^{O-C}r$$

where:

${}_{t \rightarrow t+1}^{C-C}r$ – close-close rate of return for sessions t and $t+1$,

${}_{t \rightarrow t+1}^{OV}r$ – overnight rate of return between sessions t and $t+1$,

${}_{t+1}^{O-C}r$ – open-close rate of return for sessions $t+1$,

${}_{t \rightarrow t+1}^{O-O}r$ – open-open rate of return for sessions t and $t+1$,

${}_t^{O-C}r$ – open-close rate of return for sessions t .

The open-open and close-close returns are influenced by the events and information flowing into the market during sessions (open-close rate of return) and between sessions (overnight). However, since trading can be thought of as a continuous-time process, it is also natural to consider returns over other than daily intervals. Recently, some interest has been developed into dividing daily returns into overnight (close-to-open) returns and daytime returns (Gooijer, Dicks, & Gatarek, 2009). Macroeconomic events and information published by companies affect the opening prices at the next day session, which translates into overnight interest rates. In technical analysis, higher or lower opening of the price of a given financial instrument at the next session (in relation to the last closing price) is called a price gap and is the subject of investor studies (Dahlquist & Bauer, 2012, pp. 71-106; Tam, 2007, pp. 192-207). There is considerable empirical evidence that return dynamics differ over non-trading periods and trading periods (Cliff, Cooper, & Gulen, 2008; French & Roll, 1986; George & Hwang, 2001; Hasbrouck, 1991, 1993; Lockwood & Lin, 1990; Madhavan, Richardson, & Roomans, 1997). A great number of models have been proposed to quantify this phenomenon, often using equities traded on a particular stock market, e.g., Oldfield & Rogalski (1980) and Hong & Wang (2000). The information revealed in consecutive overnight and day-time returns can also be employed for prediction. For example, predicting daytime volatility of stock prices based on the preceding overnight returns. Therefore, it seems appropriate to analyze the distribution of rates of return other than just close-close.

The hypothesis H_0 was formulated as follows: the distribution of the analyzed index returns is a normal distribution. In turn the alternative hypothesis H_1 takes the following form: the distribution of the analyzed index returns does not follow a path of a normal distribution.

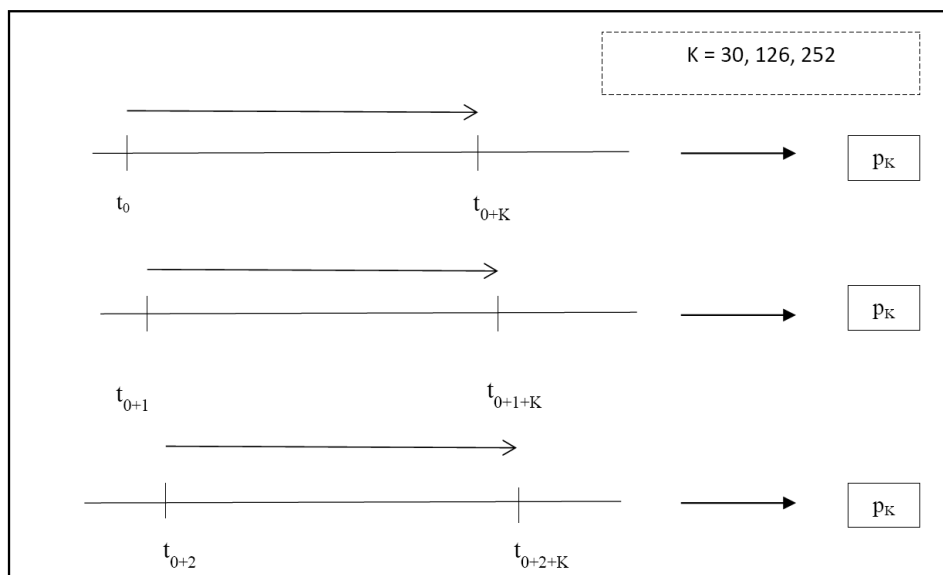
Verification of statistical hypotheses was conducted with the use of the following five statistical tests: Jarque–Bera (JB), Lilliefors (L), Cramer von Mises (CVM), Watson (W) and Anderson–Darling (AD). Each of them adopts slightly different assumptions, which influence the strength of individual tests. The powers of individual tests are also different. For example Anderson–Darling test is considered as a modification of Cramer von Mises test. It differs from the CVM in such a way that it gives more weight to the tail of the distribution (Farrel & Rogers-Stewart, 2006).

According to Razali & Yap (2011) the most powerful test is Shapiro–Wilk, followed by Anderson–Darling test, Lilliefors and Kolmogorov–Smirnov. However, the power of all four testis is still low for small sample size. Farrel & Rogers-Stewart (2006) reported that the simulated power for all tests increased as the sample size and the significance level increased.

In all analyzed cases, the p-value was calculated. If the p-value is less than or equal to 0.05, then the hypothesis H_0 is rejected in favor of the hypothesis H_1 . Otherwise, there is no reason to reject hypothesis H_0 . P-value is a common measure combining the results obtained with the application of various tests and allowing them to be compared.

In the second part, the hypothesis of the normality of daily returns for six indexes (CAC40, DAX, DJIA, FTSE250, NIKKEI225 and S&P500) were verified in the annual time horizons for 2013-2016. For the DJIA index the normality of daily return in 28 upward and downward waves (bull and bear market) was verified with the use of the succeeding tests: Jarque–Bera, Kolmogorov–Smirnov, Lilliefors, Cramer von Mises, Watson and Anderson–Darling. Part two of the study can be considered as an introduction to the third part.

In the third part of the analysis, verification of the hypothesis of normal distribution of returns was carried out according to the following scheme. Parameter p was calculated at the moment K , i.e. for the first K trading session of the analyzed index on the Warsaw Stock Exchange. If t_0 is the date of the first quotation of the index on the Warsaw Stock Exchange, then the parameter p_k was calculated for the following sessions: $t_0, t_{0+1}, t_{0+2}, \dots, t_{0+K}$. The next parameter p_k was determined for the K session time horizon, but moved forward by one session, i.e. for sessions held at times: $t_{0+1}, t_{0+2}, \dots, t_{0+K+1}$. Similarly, the value of the p_k parameter for the remaining K series sessions was computed, i.e., until the last session in the time frame (31.06.2017) – Figure 1. For all analyzed indexes, the p_k value was determined with the use of the following tests: Jarque–Bera, Shapiro–Wilk and D’Agostino–Pearson (first degree of freedom), as well as for three different time horizons of K : 30, 126 and 252 sessions (second degree of freedom) and for four types of interest rates: $C-C$, $O-O$, $O-C$ and overnight (third degree of freedom). The next step of the research was to provide statistics for each of the analyzed indexes, which include, in particular, the frequency for a given K value and the type of test when there was no reason to reject the null hypothesis. As a result of this procedure, the frequency of $p > 0.05$ is calculated for each of the tested returns, for different K and for different statistical tests.

Figure 1. Determining the p parameter in K session time horizon

Source: Author's own calculations.

Since parameter p can be treated as the probability that the analyzed distribution can be regarded as a normal distribution, and since a higher p value in a given distribution is more similar to the normal distribution, then the parameter p can be used to create a ranking list of indexes. This ranking accounts for proximity of the distribution of the index returns relative to the normal distribution. Such an index ranking was compiled for different: types of returns (C-C, O-O, C-O and overnight), K values ($K = 30$, $K = 126$ and $K = 252$ sessions) and the types of statistical tests (Jarque–Bera, Shapiro–Wilk and D’Agostino–Pearson). In the next step, for the given rates of return and for the given value of K , the sum of the ranking of an analyzed index was calculated according to the following equation:

$$S_{I+II+III} = S_I + S_{II} + S_{III},$$

where:

S_I – position in the ranking of a given index for Jarque–Bera test,

S_{II} – position in the ranking of a given index for Shapiro–Wilk test,

S_{III} – position in the ranking of a given index for D’Agostino–Pearson test.

As a result 12 rankings were obtained: (4 rates of return: C-C, O-O, O-C and overnight) x (3 investment horizons K: 30, 126 and 252 sessions). Then on the basis of these 12 ratings, the following sum of the ratings was calculated for each of analyzed indexes:

$$S_{I+\dots+XII} = {}^{K=30}_{C-C}S + {}^{K=30}_{O-O}S + {}^{K=30}_{O-C}S + {}^{K=30}_{OV}S + {}^{K=100}_{C-C}S + {}^{K=100}_{O-O}S + \\ + {}^{K=100}_{O-C}S + {}^{K=100}_{OV}S + + {}^{K=252}_{C-C}S + {}^{K=252}_{O-O}S + {}^{K=252}_{O-C}S + {}^{K=252}_{OV}S,$$

where:

K_XS – position of a given index in the ranking for a specific time horizon K ($K = 30, 126$ and 252 sessions) and return type X (C-C, O-O, O-C and overnight).

The sums $S_{I+\dots+XII}$ for each of analyzed indexes were used in the process of a global ranking construction.

The main hypothesis of the analysis has been formulated as follows: in long time intervals, equity index returns distributions are not normal distributions. As a long time interval, investment horizon covering several years was assumed. In turn, the secondary hypothesis of the research may be expressed as follow: in the shorter investment horizons, the distribution of equity indexes returns may be normal. The auxiliary hypothesis can also be written in a different way: returns of equity indexes are serially normal.

4. Research results

4.1. Verification of the normal distribution hypothesis of stock exchange index returns from their first publishing date until 31.06.2017

In case of the daily and weekly rates or return the hypothesis H_0 was rejected in favor of the hypothesis H_1 for all analyzed types of returns (C-C, O-O, O-C and overnight).

In case of monthly rates of return there was no reason to reject the H_0 hypothesis in 42 cases. The results obtained with the use of one test were confirmed by results given by another statistical test for the following number of indexes (Table 1):

- a) C-C: 4 (FTSEMIBTEL, PSI20, SESESLCT and TOPIX),
- b) O-O: 2 (SESESLCT and TOPIX),
- c) O-C: 3 (FTSEMIBTEL, SESESLCT and TOPIX).

Table 1. The value of the parameter p for is for which the obtained results with the use of one test were confirmed by the second test (monthly rates of return)

	C-C				
Index	J-B	L	CVM	W	A-D
FTSEMIB	0.0415	0.1	0.0164	0.165	0.0074
PSI20	0	0.1	0.0784	0.0907	0.039
SESESLCT	0.4831	0.1	0.683	0.6672	0.6437
TOPIX	0	0.0767	0.0775	0.118	0.1004
	O-O				
Index	J-B	L	CVM	W	A-D
FTSEMIB	0.024	0.0107	0.0018	0.0016	0.0014
PSI20	0	0.0755	0.0362	0.0334	0.0072
SESESLCT	0.4757	0.1	0.7173	0.6995	0.6592
TOPIX	0.0005	0.1	0.2413	0.3324	0.3128
	O-C				
Index	J-B	L	CVM	W	A-D
FTSEMIB	0.1016	0.1	0.0275	0.0282	0.0307
PSI20	0	0	0	0	0
SESESLCT	0.4799	0.1	0.6998	0.6848	0.655
TOPIX	0	0.1	0.07	0.1204	0.0837

Note: Values of $p > 0.05$ marked in bold.

Source: Author's own calculations.

In the case of the overnight returns the hypothesis H_0 was rejected in the case for all analyzed indexes. The p value calculated for the index SESESLCT and the following returns: C-C, O-O and O-C, was higher than 0.05 for all implemented statistical tests but for TOPIX index the p value was lower than 0.05 just only for the Jarque–Bare test.

For quarterly rates of return the number of cases when there was no reason to reject the H_0 hypothesis was as follows (Table 2 in Appendix):

- C-C (16): BUX, FTSEMIB, IBEX35, IPC, MEXICIPC, NZX50, PSI20, RUSSEL*, SAX, SDAX, SESESLCT, SSEBSHARES, TAIEX*, TOPIX, UX, XU100,
- O-O (19): BUX, FTSEMIB, IBEX35, IPC, MEXIXIPC, NZX50, OMXTALIN, PSI20, PSEI20*, RUSSEL, SAX, SDAX, SENSEX*, SESESLCT, SSEBSHARE, TAIEX, TOPIX, UX*, XU100,
- O-C (17): BUX, FTSEMIB, IBEX35, IPC, MEXIXIPC, NZX50, PSI20, RUSSEL*, SAX, SDAX, SESESLCT, SET*, SSEBSHARE, TAIEX, TOPIX, UX, XU100,
- Overnight (4): EOE, HEX, TEXCADX, TOPIX.

With * are marked these indexes when the rejection of the H_0 hypothesis was obtained with the use of one test only.

For yearly rates of return the number of cases when there was no reason to reject the H_0 hypothesis was equal to: 55 (8), 51 (3), 52 (6), 21 (6) for C-C, O-O, O-C and overnight rates of return, respectively. The number of cases in parentheses is given when the null hypothesis was rejected by only one test (Table 4 in Appendix).

These results suggest the following conclusion: the higher the data compression (daily→weekly→monthly→quarterly→yearly), the less number of H_0 hypothesis rejections.

4.2. Verification of the hypothesis of normal distribution of returns for the following indexes: CAC40, DAX, DJIA, FTSE250, NIKKEI225 and S&P500 when the investment horizon is equal to one year and during 28 up and down waves for DJIA index

The results of testing the null hypothesis for the main equity indexes in particular years are presented in Table 5 of the Appendix. In the case of many annual periods, there was no reason to reject the hypothesis of normality of the returns distribution. If for an individual index, at least two out of six tests do not allow rejecting the null hypothesis, the distribution of returns represents a normal distribution in period of the analyzed years. Such outcomes were registered for:

- a) DJIA: O-C (2013), O-O (2013) and O-C (2013) – odd year,
- b) DAX: C-C (2015), O-O (2015) and O-C (2015) – odd year,
- c) S&P500: Overnight (2016) – even year,
- d) FTSE250: C-C (2014), O-O (2014) and O-C (2014) – even year,
- e) CAC40: O-C (2016) – even year,
- f) NIKKEI225: C-C (2013), O-O (2013 and 2014) and Overnight (2013, 2014, 2015 and 2016) – odd and even years.

The results of testing the null hypothesis for the indexes DJIA in particular bull and bear markets are presented in Table 6 of the Appendix. In the following four downward index waves there was no reason to reject the hypothesis H_0 :

1. 11.02.1966-11.10.1966 (wave nr 15),
2. 06.12.1968-26.05.1970 (wave nr 17),
3. 12.01.1973-10.12.1974 (wave nr 19),
4. 24.09.1976-02.03.1978 (wave nr 21).

The p value coefficients higher than 0.05 were observed for the following upward index movements:

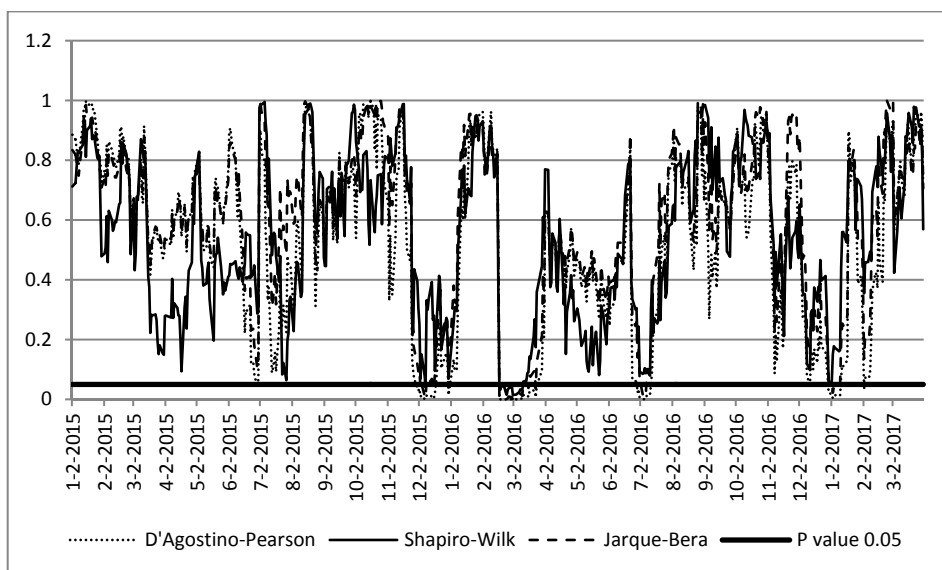
1. 11.10.1966-06.12.1968 (wave nr 16),
2. 10.12.1974-24.09.1976 (wave nr 20).

The null hypothesis was rejected for rising and falling waves, when the ends of the waves fall in the even year. All the up and down waves, for which there was no reason to rejecting the null hypothesis, were observed in the 1960s and 1970s.

4.3. Testing hypotheses for $K = 30$, $K = 126$ and $K = 252$ sessions, and related statistics

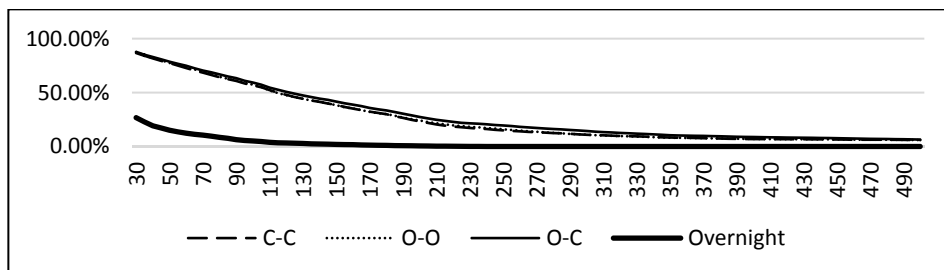
For all analyzed indexes and for three K parameters (30, 126 and 252 sessions) the following three tests were performed: Jarque–Bera, Shapiro–Wilk and D’Agostino–Pearson. The choice of $K = 30$ is based on the assumption that a sample size of about 30 elements in a t-student distribution approximates a normal distribution. In turn, $K = 252$ is approximately equal to the number of sessions per year, and $K = 126$ corresponds to the number of sessions in about 6 months. The results are shown in Figures 2-8 and in Table 2.

Figure 2. Value of parameter p for DJIA in the period 02.01.2015-31.03.2017 when carrying out three different tests and $K = 30$



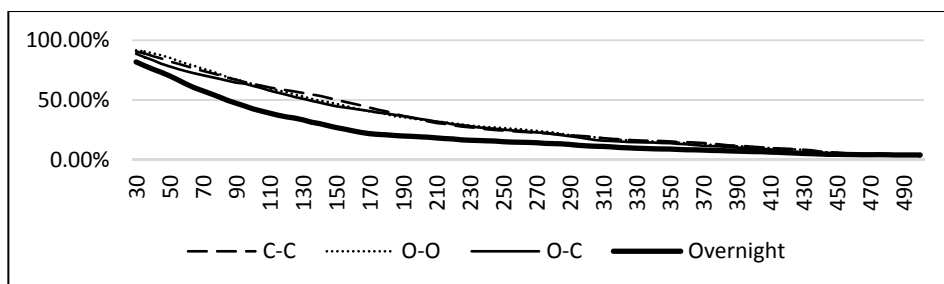
Source: Author's own calculations.

Figure 3. The percentage of cases where there was no reason for rejecting the null hypothesis for DJIA returns with the use of Jarque–Bera test, depending on K (change of K: every 5 units)



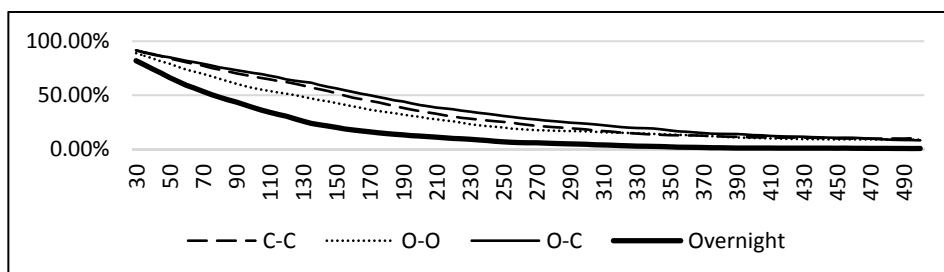
Source: Author’s own calculations.

Figure 4. The percentage of cases where there was no basis for rejecting the null hypothesis for DAX returns with the use of Jarque–Bera test, depending on K (change of K: every 5 units)



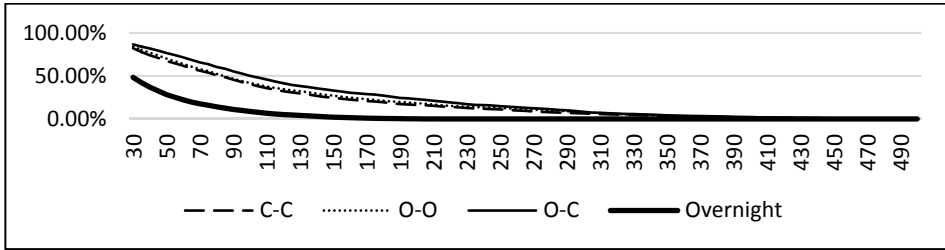
Source: Author’s own calculations.

Figure 5. The percentage of cases where there was no basis for rejecting the null hypothesis for CAC40 index returns with the use of Jarque–Bera test, depending on K (change K: every 5 units)



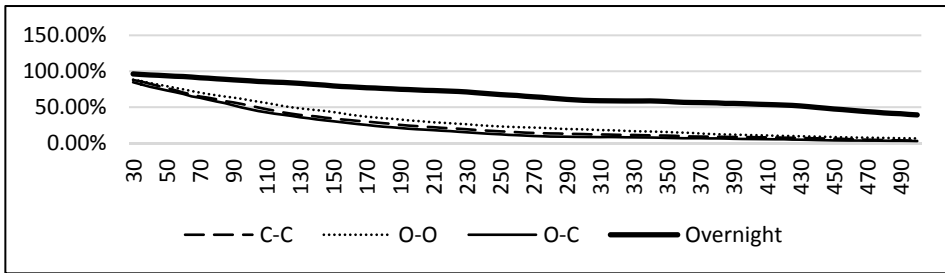
Source: Author’s own calculations.

Figure 6. The percentage of cases where there was no basis for rejecting the null hypothesis for FTSE250 index returns with the use of Jarque–Bera test, depending on K (change K: every 5 units)



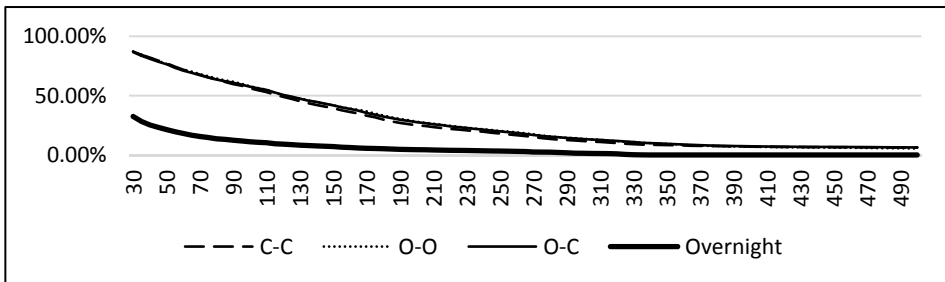
Source: Author’s own calculations.

Figure 7. The percentage of cases where there was no basis for rejecting the null hypothesis for Nikkei index returns with the use of Jarque–Bera test, depending on K (change K: every 5 units)



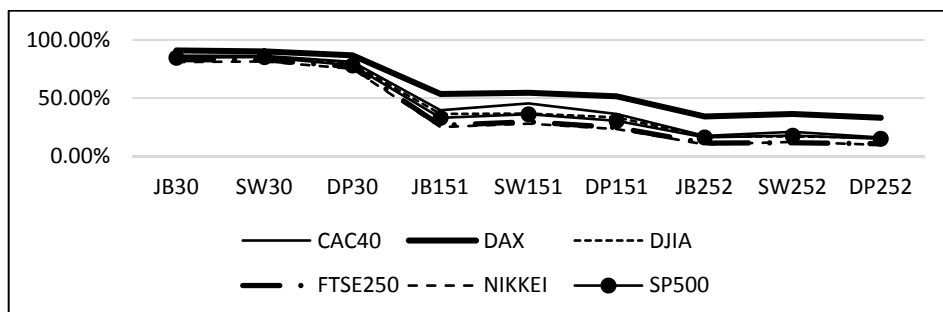
Source: Author’s own calculations.

Figure 8. The percentage of cases where there was no basis for rejecting the null hypothesis for S&P500 index returns with the use of Jarque–Bera test, depending on K (change K: every 5 units)



Source: Author’s own calculations.

Figure 9. The percentage of cases where there was no basis for rejecting the null hypothesis for four main indexes returns with the use of Jarque–Bera test, depending on K (change K: every 5 units) and C-C rates of return



Source: Author's own calculations.

An increase of the parameter K leads to a decrease in the percentage of cases when there was no reason to reject the null hypothesis. This tendency is especially noticeable in case of overnight rates of return because only in very few cases do significant events take place in the company's environment, which result in a meaningful deviation of the opening price in relation to the last closing price. In this case, a significant percentage of returns is close to zero. For C-C, O-O and O-C rates of return, a broader horizon of observation was required to increase the percentage of cases where there was no reason to reject the null hypothesis. This drift was common for all analyzed indexes (DAX, CAC40, FTSE250, DJIA and S&P500) except NIKKEI225, for which the opposite trend was noted.

For small K, the highest percentage of non-rejecting null hypothesis was observed for the DAX index, followed by CAC40, DJIA, S&P500, FTSE250 and NIKKEI225 (Figure 9). With the increase of the parameter K, this order remained stable. For K = 252 the deference in percentage of non-rejection null hypothesis between DAX and NIKKEI225 was higher than for K = 30.

The Table 2 presents index rankings due to the proximity of the distribution of returns of analyzed indexes to the normal distribution.

Table 2. Ranking of equity indexes due to the proximity of their rates of return to the normal distribution

Item	Index	30 sessions				126 sessions				252 sessions				Total
		C-C	O-O	O-C	OV	C-C	O-O	O-C	OV	C-C	O-O	O-C	OV	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	AEX	8	24	9	6	4	15	5	6	3	10	3	5	1
2	All Ordinaries	15	14	35	49	11	7	20	38	4	4	11	23	13
3	Athex Composite	53	52	46	23	54	50	44	18	53	50	42	19	46
4	BEL20	24	30	36	9	22	34	43	10	27	37	39	11	25
5	BET	55	56	59	38	55	55	57	39	55	54	55	43	55
6	Bovespa	10	5	4	62	9	5	4	60	6	5	5	57	15
7	BUX	49	42	15	26	44	40	22	30	41	39	21	33	35
8	CAC40	29	36	7	7	29	36	7	8	26	32	6	9	15
9	CDAX	6	11	24	2	5	17	31	2	12	29	39	2	8
10	DAX	7	2	27	3	7	4	24	3	7	6	29	3	5
11	DJCA	25	27	44	32	20	25	39	28	16	23	37	29	29
12	DJIA	34	21	33	60	35	20	29	54	37	19	26	45	38
13	DJTA	43	43	48	50	43	39	46	42	39	34	40	37	46
14	DJUA	48	35	50	57	47	37	47	57	45	38	47	50	52
15	EOE	4	6	17	11	3	6	16	9	6	8	14	8	2
16	FSE100	50	35	31	47	50	42	36	51	50	45	43	53	50
17	FTSE250	28	8	35	13	30	9	30	12	35	12	27	13	18
18	FTSEMIBTEL	19	23	18	47	17	23	20	50	16	25	18	54	27
19	HANG SENG	23	19	26	5	18	19	27	5	17	17	19	6	11
20	HEX	58	57	61	51	58	57	62	55	60	59	62	60	61
21	IBEX	14	18	25	63	19	22	33	61	21	19	36	61	32
22	ICEX	37	46	37	55	37	42	34	52	32	40	31	47	44
23	IPC	60	61	53	40	59	61	52	43	57	58	52	46	57
24	IPSA	52	55	52	33	54	56	54	36	54	56	56	36	54
25	JCI	48	37	12	10	41	32	9	7	31	24	9	7	22
26	KLCI	9	10	12	12	13	16	15	14	19	22	20	21	9
27	KOSPI	39	31	47	54	42	38	48	46	43	43	50	41	49
28	MDAX	30	28	43	56	31	27	45	50	33	30	46	40	42
29	MERVAL	51	51	49	25	51	52	50	24	48	51	48	22	49
30	MEXICIPC	19	17	28	39	15	14	18	37	14	13	13	32	21
31	MICEX	54	53	56	4	52	52	55	4	52	49	54	4	43
32	NASDAQ100	16	9	9	59	12	8	8	59	8	7	7	55	20
33	NASDAQCOMP	59	60	58	61	60	58	58	62	60	58	57	62	62
34	NIKKEI225	14	15	19	29	16	13	18	23	13	9	12	16	10
35	NZX50	56	54	55	43	56	54	56	45	56	55	58	50	56
36	OMXRIGA	64	64	62	48	64	64	61	57	64	63	59	58	63
37	OMXSTOCKOLM	21	12	1	37	23	19	12	32	28	20	24	26	19
38	OMXTALIN	46	47	45	42	50	47	49	42	51	52	51	42	53
39	OMXVILNUS	36	48	42	19	33	46	37	21	30	42	26	26	37
40	OSE	26	26	29	19	33	29	32	15	34	31	30	15	24
41	PSEI	44	44	54	37	45	44	53	42	47	45	50	51	51

Table 2 cont.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
42	PSI20	31	45	40	17	37	48	40	17	40	49	42	19	39
43	PX50	41	25	38	34	40	26	38	32	42	27	34	29	37
44	RTS	65	65	65	53	65	65	65	59	65	65	65	59	65
45	RUSSEL	42	51	51	15	48	53	51	20	49	53	53	27	47
46	SAX	33	23	10	27	28	21	10	22	23	11	9	21	16
47	SDAX	45	38	16	30	46	34	14	35	46	35	18	39	33
48	SEECOM	17	29	20	24	24	29	23	29	36	34	32	38	28
49	SENSEX	57	58	57	41	57	59	59	48	58	61	61	56	58
50	SESESLCT	63	63	64	58	63	64	64	63	63	64	64	63	65
51	SET	61	59	60	45	61	61	60	48	61	60	60	52	59
52	SMI	27	49	13	31	26	49	13	27	24	49	15	24	30
53	SOFIX	35	33	41	28	38	44	42	33	44	41	45	34	41
54	SP500	5	20	14	14	7	24	11	26	9	28	18	35	12
55	SPTSXCOM	40	42	2	16	34	31	3	13	23	21	4	10	17
56	SSESHARE	62	62	63	45	62	62	63	44	62	63	63	45	60
57	Straits Times	38	40	40	20	39	45	42	19	38	46	44	19	40
58	TAIEX	3	14	5	8	8	11	6	11	11	15	10	14	3
59	TECDAX	2	1	3	52	2	1	2	53	2	3	2	50	7
60	TOPIX	21	33	21	22	22	35	27	25	26	37	28	30	26
61	TSE300	12	3	32	1	14	2	28	1	19	2	33	1	6
62	UK100	11	16	30	64	11	13	27	64	10	14	24	64	31
63	UX	1	4	6	35	1	3	1	34	1	1	1	31	4
64	WIG	22	7	23	65	25	10	35	65	29	16	35	65	34
65	XU100	33	40	23	21	27	31	22	16	21	26	22	13	23

Source: Author's own calculations.

For example, for $K = 30$ sessions and C-C rates of return the first three places were ranked as follows: UX, TECDAX and TAIEX, while the last three in order: SESESLCT, OMXRIGA and RTS. In turn, in the total ranking, the top three indexes were: AEX, EOE and TAIEX, and the last three: OMXRIGA, RTS and SESESLCT.

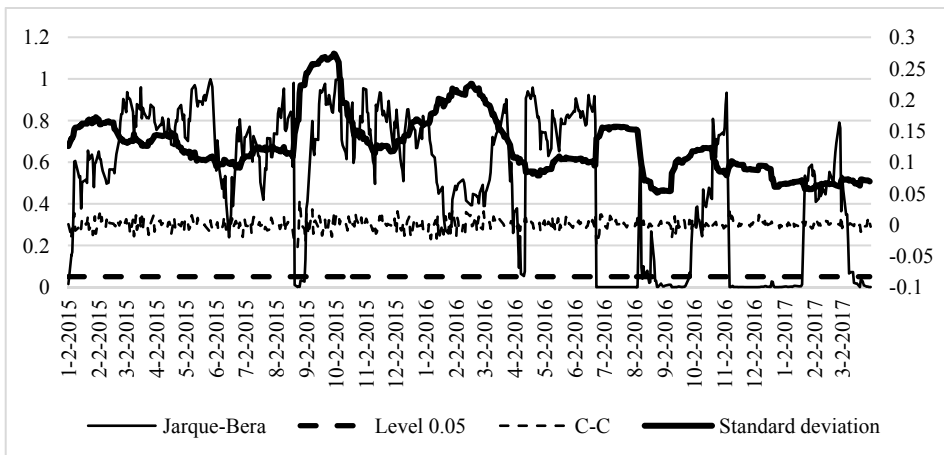
5. Research findings discussion

The values of parameters p , calculated with the use of tests of Cramer–Von Mises and Anderson–Darling, in some cases were similar, but in many – they were different. For example, for data included in Table 1, the absolute values of the difference of both statistics greater than 0.005 were recorded in the following number of cases: 16 (C-C), 16 (O-O), 14 (O-C) and 5 (Overnight).

For daily returns, calculated in particular years for six main world stock indexes, there was no reason to reject the null hypothesis in individual years, and in most cases generally for the following rates of return: C-C, O-C and O-O. For analyzed up and down waves of the DJIA index (daily data), individual cases were recorded when there were no reason to reject the null hypothesis. However, in the majority of analyzed years (2013-2016), as well as for up and down DJIA waves, the zero hypothesis was rejected.

According to the total ranking of equity indexes due to the proximity of their rates of return to the normal distribution the best and the worst performing indexes were AEX and OMX RIGA. The indexes of the most developed global stock exchanges classified in the total ranking in the following positions: DAX (5), TSE300 (6), NIKKEI225 (10), S&P500 (12), CAC40 (15), FTSE250 (18), NASDAQ100 (20) and DJIA (38). So in the case of several global indexes, e.g., CAC40, FTSE250, NASDAQ100 and DJIA, the distribution of analyzed rates of return is far from normal. The analysis of rankings for overnight and close-close returns and periods of 30-, 126-, and 252-sessions proved that the indexes of the developed markets dominated the top places (DAX, S&P500, NASDAQ100, UK100, All Ordinaries, NIKKEI225). The indexes of emerging markets placed at the end of the ranking (RTS, OMXRIGA, SESESLCT, SSEBSHARE), although a few exceptions could be given. In the case of remaining return rates, no similar relationship can be found. The created ranking can be applied in the investment decisions by investors using transaction systems based on the distribution of return rates.

Figure 10. The p-parameter chart for the DJIA index, $K = 30$, the Jarque–Bera test, the return rate (C-C) and the annualized standard deviation (p-value and level 0.05- left scale, C-C return and standard deviation – right scale)



Source: Author's own calculations.

Analysis of the results obtained for $K = 30$ sessions concludes that for such short time interval, a sharp index change leads to a violent decrease in the value of parameter p . This process is illustrated in Figure 8, which includes the index DJIA returns (C-C), parameter p and annualized standard deviation of returns. For example, with a strong increase in volatility on 19.08.2015, the value of p dropped below the trigger value of 0.05. Explanation of the decrease in the value of parameter p below 0.05 for $K = 126$ and $K = 252$ sessions becomes more complex and requires further investigation.

6. Conclusions

Some of the conducted calculations prove unequivocally that the distribution of daily returns of equity indexes is not normal distribution, thus confirming the results obtained by other researchers, such as Kendall (1953), Fama (1976), Barunik et al. (2010). This remark applies to C-C rates of return. The paper also shows that the distribution of the remaining daily returns, e.g., O-O, C-O and overnight, calculated for the analyzed equity indexes do not follow a normal distribution. This is one of the scarce studies (if not the only one), in which rates of return other than close-close were analyzed, and the first one regarding Polish index WIG. In addition, other papers focused only on one or two statistical tests, while five different tests were implemented in this paper.

From these results, the following conclusion can be drawn: the higher the data compression (from daily to yearly), the fewer H_0 hypothesis rejections. In the case of overnight returns (quarterly and annual data) the smallest number of cases were observed, when there were no reasons to reject the null hypothesis. For the daily, weekly and monthly overnight rates of return, the null hypothesis was rejected for all analyzed indexes.

With the use of the parameter p , a stock index ranking was also created for time horizons of $K = 30$, $K = 126$, and $K = 252$ sessions. A stock index ranking is possible because of approximating the distribution of index returns with a normal distribution. As such, it was found that the position of the index in the ranking is not dependent on the date of its first publication, and hence on the number of rates of return possible to calculate for analyzed index, but mainly on the distribution of the index rates of return. The higher the position in the created ranking, the closer the distribution of return rates of a given index to the normal distribution (taking into account the ranking criteria).

Furthermore, the data suggests that the distribution of returns can be normal only in given time intervals. Time intervals can be set as individual years or up and down waves. The obtained results are consistent with those of Piasecki & Tomasik (2013, pp. 34-89) who proved the normal distribution of returns in certain upward and downward price movements on the Polish equity market.

According to the obtained results, the distribution of the rates of return in the majority of cases are different than normal, which question the possibility of unreflective implementation in practice of economic models such as CAPM and its derivatives, Black–Scholes options valuation, portfolio theory and efficient market hypothesis, especially in long time horizons. For the short time horizons, in most cases, the distribution of rates of return was close to normal, which empowers investors to use the above-mentioned models. The results constitute a voice in the ongoing discussions dedicated to the effectiveness of financial markets, and thus the possibility of effective investment with the use of technical and fundamental analysis.

The limitations of obtained results, relate to different time horizons, for which the index rates of return were calculated.

Futures studies, using the methods presented in this paper, should be conducted for commodities and the FX market in order to determine the normality of rate of return distributions.

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Appendix

Table 3. The value of the parameter p for quarterly equity index rates of return

Index	Date of the first index publishing	C-C				O-O				O-C				Overnight								
		J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	
<i>I</i>	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
AEX	03.01.1983	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ALL Ordinaries	01.01.1900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Athex Com	02.01.1987	0	0.0011	0	0	0	0	0.0001	0	0	0	0	0.0002	0	0	0	0	0	0	0	0	0
BEL20	02.01.1991	0	0.0339	0.0077	0.0162	0.0038	0	0.0105	0.0021	0.0034	0.0012	0	0.0054	0.0035	0.0066	0.0021	0	0	0	0	0	0
BET	31.10.2000	0.0214	0.0116	0.0048	0.0037	0.0036	0.0361	0.0096	0.0024	0.0015	0.0021	0.0265	0.0246	0.003	0.0021	0.0024	0	0	0	0	0	0
Bovespa	12.07.1989	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bux	02.01.1991	0.0008	0.1	0.0869	0.0701	0.0406	0	0.1	0.0954	0.0791	0.0354	0.0004	0.0952	0.1099	0.091	0.0436	0	0	0	0	0	0
CAC40	08.01.1965	0	0.0001	0	0	0	0.0013	0	0	0	0	0	0.0002	0	0	0	0	0	0	0	0	0
CDAX	15.03.2004	0.0074	0.0266	0.0156	0.0272	0.0158	0.0005	0.0117	0.0055	0.0101	0.0055	0.0062	0.0076	0.0143	0.0257	0.0133	0	0.0088	0	0	0	0
DAX	28.09.1959	0	0.0022	0	0	0	0	0.0005	0	0.0001	0	0	0.0007	0.0001	0.0001	0	0	0	0	0	0	0
DICA	23.12.1980	0	0.0002	0	0.0001	0	0	0.0002	0	0	0	0	0	0	0.0001	0	0	0	0	0	0	0
DJIA	02.01.1900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DJTA	02.01.1929	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DJUA	02.01.1929	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EOE	02.01.1995	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0001	0	0.1928	0.1987	0.1194	0
FTSE100	22.10.1992	0.0288	0.0084	0.0007	0.0007	0.0004	0.0309	0.0115	0.0005	0.0005	0.0004	0.0389	0.0258	0.0069	0.0009	0.0007	0	0	0	0	0	0
FTSE250	31.12.1985	0	0.0286	0.003	0.0062	0.0007	0.0003	0.003	0.0084	0.015	0.0021	0	0.003	0.0017	0.0033	0.0004	0	0	0	0	0	0
FTSEMIB	02.01.1999	0.3184	0.1	0.1071	0.1262	0.1145	0.2693	0.0655	0.0451	0.0466	0.0375	0.3643	0.0887	0.1168	0.1325	0.1105	0	0	0	0	0	0
HANGSENG	24.11.1969	0	0.0001	0	0	0	0	0.0033	0	0	0	0	0.0028	0.0001	0	0	0	0	0	0	0	0
HEX	02.01.1995	0	0.0016	0.0012	0	0	0.0003	0.0009	0.0006	0.0003	0.0006	0.0004	0.0017	0.001	0.0005	0.0008	0.0529	0.0217	0.0145	0.0334	0	0
IBEX35	05.01.1987	0.3011	0.1	0.2513	0.2778	0.2718	0.1747	0.1	0.1625	0.1594	0.1402	0.4314	0.1	0.3909	0.3899	0.3683	0	0	0	0	0	0
ICEX	31.12.1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IPC	08.11.1991	0.9324	0.1	0.9658	0.9601	0.9017	0.8824	0.1	0.9774	0.9747	0.9513	0.9648	0.1	0.9729	0.9677	0.9115	0	0	0	0	0	0
IPSA	02.01.1987	0.024	0.0043	0.0158	0.0156	0.0168	0.0054	0.0335	0.0163	0.0171	0.0171	0.0174	0.0084	0.0146	0.0142	0.0162	0	0	0	0	0	0

Table 3 cont.

<i>I</i>	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
SP500	02.01.1900	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SPTSXCOMP	03.01.1961	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SSESHARE	04.01.2000	0.9061	0.1	0.3675	0.5428	0.3607	0.9655	0.1	0.4023	0.3708	0.4532	0.9032	0.1	0.3316	0.3085	0.3486	0	0	0	0	0	0
SSECOMP	19.12.1990	0	0.0073	0.0005	0.0004	0.0004	0	0.0036	0.0011	0.0008	0.0006	0	0.0011	0.0004	0.0003	0.0002	0	0	0	0	0	0
STRAITSTIMES	28.12.1987	0	0.0012	0.0001	0	0	0	0.0116	0.0002	0.0001	0.0001	0	0.0027	0.0001	0	0	0	0	0	0	0	0
TAIEX	05.01.1995	0.1773	0.0231	0.0177	0.0145	0.0194	0.3776	0.1	0.0546	0.0488	0.057	0.2156	0.087	0.0237	0.0201	0.0223	0	0	0	0	0	0
TECDAX	16.09.1999	0	0.0001	0	0	0	0	0.0002	0	0	0	0	0.0001	0	0	0	0.649	0.0773	0.0275	0.0186	0.0391	
TOPIX	22.10.2001	0.5106	0.1	0.6505	0.6227	0.6598	0.4606	0.1	0.4957	0.4951	0.53	0.4987	0.1	0.6941	0.6889	0.7184	0.3997	0.1	0.2379	0.2578	0.3101	
TSE300	15.08.1989	0	0.0022	0.0003	0.0007	0.0001	0	0.0117	0.001	0.0024	0.0003	0	0.01	0.0008	0.0016	0.0003	0	0	0.0001	0	0.0001	
UK100	13.11.1935	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
UX	03.11.1997	0.1311	0.0799	0.0211	0.0139	0.0263	0.3538	0.0487	0.0298	0.0207	0.042	0.3577	0.0559	0.0508	0.0371	0.0548	0	0	0	0	0	0
WIG-M	16.04.1991	0	0.0009	0.0002	0.0002	0	0	0.0005	0.0001	0.0001	0	0	0	0.0002	0.0001	0	0	0	0	0	0	0
XU100	02.01.1990	0.0038	0.1	0.0603	0.0565	0.0686	0.0098	0.1	0.0598	0.0603	0.0568	0.0052	0.1	0.0592	0.0564	0.0565	0	0	0	0	0	0

Note: Values of $p > 0.05$ marked in bold.

Source: Author's own calculations.

Table 4 cont.

<i>i</i>	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
MEXICIPC	20.12.1993	0.8358	0.1	0.683	0.6284	0.5925	0.7797	0.1	0.7445	0.6963	0.6765	0.8348	0.1	0.687	0.633	0.5824	0	0	0	0	0
MICEX	27.09.1997	0.0924	0.1	0.1559	0.1339	0.1009	0.6609	0.1	0.3521	0.3242	0.3665	0.1441	0.1	0.2237	0.1988	0.1542	0	0	0	0	0
NASDAQ 100	01.10.1985	0.513	0.0022	0.011	0.0083	0.0122	0.503	0.022	0.0071	0.0051	0.0085	0.506	0.0023	0.0088	0.0064	0.0099	0.0005	0	0	0	0
NASDAQCOMP	03.01.1938	0.0036	0.1	0.0138	0.0199	0.0062	0.0039	0.1	0.187	0.0256	0.008	0.0015	0.01	0.0122	0.0185	0.0046	0.0009	0.0101	0.0007	0.0004	0.0004
NIKKEI225	01.03.1914	0.1024	0.1	0.0515	0.0405	0.0588	0.2058	0.1	0.0621	0.0498	0.0859	0.1144	0.1	0.0353	0.0264	0.0457	0	0	0	0	0
NZX50	03.01.2001	0	0.0966	0.0234	0.037	0.0071	0.0001	0.1	0.0252	0.0388	0.0083	0	0.0919	0.0232	0.0365	0.0071	0	0	0	0	0
OMXRIGA	03.01.2000	0.0658	0.1	0.2997	0.3343	0.1337	0.1343	0.1	0.2358	0.2794	0.0947	0.0658	0.1	0.3013	0.3361	0.1347	0.0754	0.0001	0	0	0
OMXSTOCKHOLM	30.09.1986	0.2116	0.1	0.1538	0.2025	0.156	0.2298	0.0634	0.0651	0.0887	0.0844	0.1863	0.1	0.1484	0.2044	0.1464	0.002	0.0017	0.0005	0.0003	0.0006
OMXTALIN	03.01.2000	0.0011	0.1	0.0448	0.0718	0.0231	0.0011	0.1	0.434	0.6093	0.0231	0.0013	0.1	0.0454	0.0728	0.0239	0.222	0.0055	0.0037	0.0032	0.0079
OMXVILNIUS	01.10.2000	0	0.0866	0.0115	0.007	0.0079	0	0.0333	0.0219	0.0145	0.0125	0	0.0249	0.0104	0.0063	0.0074	0	0	0	0	0
OSE	03.01.1983	0.0611	0.1	0.5837	0.6847	0.3559	0.0446	0.1	0.5075	0.629	0.3254	0.0618	0.1	0.5919	0.6903	0.3585	0	0	0	0	0
PSEI	02.01.1986	0.7539	0.1	0.2912	0.2619	0.2948	0.9439	0.1	0.5634	0.5196	0.5678	0.7316	0.1	0.5017	0.2723	0.3015	0	0	0	0	0
PSI20	31.12.1992	0.3487	0.21	0.3627	0.3869	0.3359	0.443	0.1	0.4444	0.4762	0.4716	0.3576	0.1	0.4514	0.4845	0.4135	0.5476	0.0283	0.051	0.0428	0.0675
PX50	07.09.1993	0.1091	0.1	0.6604	0.7452	0.5061	0.2932	0.1	0.7693	0.825	0.6173	0.0836	0.1	0.6338	0.7361	0.4926	0.9651	0.1	0.3207	0.2868	0.4311
RTS	01.09.1995	0.0152	0.1	0.0449	0.0717	0.0313	0.0212	0.1	0.0796	0.1314	0.046	0.0126	0.1	0.0448	0.03	0.03	0.0186	0.001	0.0006	0.0007	0.0005
RUSSSEL	22.10.2001	0.4265	0.1	0.3788	0.4332	0.3769	0.4536	0.1	0.4949	0.5462	0.4828	0.4505	0.1	0.4583	0.517	0.4509	0.0405	0.0101	0.0008	0.001	0.0004
SAX	03.07.1995	0.04351	0.1	0.6193	0.602	0.4902	0.4391	0.1	0.508	0.5015	0.4491	0.4538	0.1	0.5842	0.5716	0.5043	0.0442	0	0.0001	0.0001	0.0001
SDAX	15.03.1999	0.2871	0.0862	0.0779	0.0969	0.0887	0.2926	0.0616	0.0699	0.0861	0.0836	0.2855	0.0766	0.0749	0.093	0.0877	0.5323	0.1	0.4582	0.4342	0.3701
SENSEX	03.04.1979	0.0626	0.1	0.9053	0.9346	0.7964	0.4826	0.1	0.8978	0.9138	0.831	0.4691	..1	0.9684	0.9762	0.8859	0	0	0	0	0
SESSELCT	02.01.2003	0.7815	0.1	0.7622	0.7225	0.7293	0.7536	0.1	0.949	0.9411	0.9108	0.7836	0.1	0.7711	0.7334	0.749	0	0.0013	0	0	0
SET	02.07.1987	0.0953	0.1	0.5059	0.4725	0.5531	0.9136	0.1	0.4802	0.4527	0.5086	0.9504	0.1	0.5059	0.4737	0.5351	0.3412	0.0249	0.0006	0.0003	0.0007
SMI	01.07.1988	0.0561	0.1	0.2456	0.2703	0.2974	0.5876	0.1	0.1554	0.1667	0.188	0.6257	0.1	0.2568	0.2743	0.3317	0.0164	0.0003	0.0008	0.001	0.0009
SOFIX	26.11.2001	0	0.0136	0.0037	0.0038	0.0021	0	0.0183	0.0058	0.0064	0.003	0	0.0132	0.0035	0.0035	0.002	0.8339	0.0003	0.0005	0.0002	0.0017
SF500	02.01.1900	0	0.0707	0.0104	0.0291	0.0048	0	0.0322	0.009	0.0246	0.0042	0	0.0736	0.0114	0.0323	0.0048	0	0	0	0	0
SPTXCOMP	03.01.1961	0.0121	0.1	0.02061	0.3356	0.1396	0.0872	0.1	0.6121	0.7262	0.4567	0.0393	0.1	0.4637	0.5996	0.2751	0	0	0	0	0
SSESHARE	04.01.2000	0.9708	0.1	0.5502	0.5071	0.5143	0.8614	0.1	0.8467	0.8272	0.7696	0.9691	0.1	0.5573	0.5121	0.5151	0.3781	0.0198	0.0318	0.013	0.0178
SSECOMP	19.12.1990	0.6804	0.0624	0.0768	0.0609	0.0696	0.9178	0.1	0.1377	0.1164	0.1222	0.6857	0.0585	0.0634	0.0492	0.0599	0.0001	0.0217	0.0009	0.0005	0.0008
STRAITSTIMES	28.12.1987	0.8503	0.1	0.9707	0.9663	0.9272	0.8552	0.1	0.9932	0.9931	0.9749	0.8734	0.1	0.9819	0.9792	0.9481	0.3392	0.0651	0.0387	0.0454	0.0488
TAIEX	05.01.1995	0.4281	0.0042	0.0092	0.0075	0.0144	0.4636	0.0075	0.0486	0.0493	0.0674	0.3551	0.006	0.0165	0.0153	0.0227	0.7351	0.0886	0.0174	0.0123	0.0285
TECDAX	16.09.1999	0.1056	0.014	0.0041	0.0054	0.0041	0.2498	0.0446	0.0085	0.0095	0.0108	0.1016	0.0141	0.0039	0.0051	0.004	0.5781	0.0214	0.0453	0.0435	0.0731
TOPIX	22.10.2001	0.4627	0.1	0.3271	0.3264	0.3583	0.7187	0.1	0.4759	0.4683	0.5121	0.448	0.1	0.2819	0.2817	0.3094	0.7078	0.1	0.2528	0.242	0.3326

Table 4 cont.

<i>I</i>	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
TSE300	15.08.1989	0.019	0.1	0.2172	0.2981	0.1627	0.0753	0.0228	0.1111	0.1478	0.1119	0.018	0.0908	0.1857	0.2523	0.1428	0.1061	0.1	0.6222	0.6157	0.5061
UK100	13.11.1935	0.2238	0.1	0.1423	0.1382	0.1322	0.2783	0.1	0.1709	0.1646	0.1659	0.3385	0.1	0.1692	0.1651	0.1609	0	0	0	0	0
LUX	03.11.1997	0.3761	0.1	0.494	0.5381	0.5114	0.5452	0.1	0.2958	0.3292	0.264	0.459	0.1	0.5141	0.5442	0.5395	0	0	0	0	0
WIG	16.04.1991	0	0.0001	0.0001	0.0001	0	0	0.0001	0.0001	0	0	0	0.0001	0.0001	0.0001	0	0.1013	0.0067	0.0016	0.0019	0.0021
XU100	02.01.1990	0.223	0.1	0.1443	0.1841	0.1124	0.2477	0.0958	0.1374	0.1728	0.1074	0.2558	0.1	0.2158	0.276	0.1726	0.0001	0.0028	0	0	0

Note: Values of $p > 0.05$ marked in bold.

Source: Author's own calculations.

Table 5. Results of testing the null hypothesis for each year (in the period of 2013-2016)

Index	Year	C-C						O-O						O-C						Overnight									
		J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D	J-B	L	CVM	W	A-D			
DJIA	2013	0	0.1	0.0626	0.0612	0.0364	0	0.1	0.0796	0.0763	0.533	0	0.1	0.053	0.501	0.0299	0	0	0	0	0	0	0	0	0	0	0	0	
	2014	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	2015	0	0.0697	0.0159	0.0107	0.0205	0	0.071	0.0128	0.0082	0.0149	0	0.1	0.0139	0.0094	0.0158	0	0	0	0	0	0	0	0	0	0	0	0	
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	2013	0	0.0419	0.017	0.019	0.005	0	0.0083	0.005	0.0045	0.0019	0	0.0918	0.0075	0.0079	0.0023	0.0005	0	0	0	0	0.1	0.0139	0.0108	0.0117	0	0	0	0
	2014	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SP-500	2015	0	0.0073	0.0007	0.0003	0.0008	0	0.0085	0.0011	0.0005	0.0013	0.0082	0.0012	0.0007	0.0011	0	0	0	0	0	0	0	0.1	0.0091	0.0062	0.0048	0	0	
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.03	0.0206	0.0135	0.0275	
	2013	0.0004	0.0075	0.004	0.0025	0.0009	0.0038	0.0042	0.0005	0.0004	0.0002	0	0	0.0017	0.0001	0.0001	0	0	0	0	0	0.0046	0.0385	0.1472	0.1323	0.1836	0	0	
	2014	0.0351	0.0009	0.0001	0	0.0001	0.1517	0.0209	0.0137	0.0126	0.0123	0.01945	0.0015	0.0035	0.0031	0.0042	0	0	0	0	0.0146	0.0001	0.0001	0.0001	0.0001	0	0	0	
	2015	0.2063	0.1	0.0707	0.062	0.1028	0.0349	0.0391	0.0504	0.0562	0.0355	0.0265	0.048	0.0946	0.0132	0.1034	0	0	0	0.0051	0.0009	0.0004	0.0005	0	0	0	0	0	
	2016	0	0.005	0.0007	0.0006	0.0005	0	0	0	0	0	0	0	0	0.0051	0.0009	0.0004	0.0005	0	0	0	0	0	0	0	0	0	0	
CAC40	2013	0.0001	0.0858	0.0074	0.0045	0.0058	0	0.1	0.0183	0.0209	0.0103	0	0.1	0.014	0.011	0.0084	0	0	0	0	0.0084	0	0	0	0	0	0	0	
	2014	0.0002	0.0146	0.0032	0.002	0.0023	0.0002	0.0008	0.0005	0.0003	0	0.0034	0.0028	0.0024	0.0038	0	0	0	0	0	0.004	0.0001	0	0	0	0	0	0	
	2015	0.0005	0.0326	0.0028	0.0017	0.002	0	0.00435	0.0062	0.0072	0.0023	0.0146	0.0409	0.0206	0.0303	0.0272	0	0	0	0	0.004	0.0001	0	0	0	0	0	0	
	2016	0	0.0004	0	0	0	0	0	0	0	0	0	0	0	0.1	0.1406	0.0894	0	0	0	0	0	0	0	0	0	0	0	
	2013	0	0.0768	0.0125	0.0155	0.0079	0	0.1	0.0183	0.0209	0.0103	0	0.1	0.014	0.011	0.0084	0	0	0	0	0	0	0	0	0	0	0	0	0
	2014	0.4816	0.1	0.6962	0.708	0.6796	0.4888	0.1	0.6872	0.6928	0.6745	0.4836	0.1	0.7217	0.7342	0.6991	0	0	0	0	0.0096	0.0106	0	0	0	0	0	0	
FTSE250	2015	0	0.1	0.0145	0.0096	0.0106	0	0.1	0.0138	0.0092	0.0102	0.1	0.0145	0.0096	0.0096	0.0106	0	0	0	0	0	0	0	0	0	0	0	0	
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
	2013	0	0.0786	0.0518	0.0887	0.0153	0.0068	0.1	0.3419	0.3491	0.2771	0	0.0002	0.0001	0.0001	0	0	0	0	0	0.0632	0.1	0.1423	0.1439	0.0576	0	0	0	
	2014	0.0005	0.0013	0.0001	0	0	0	0.1	0.3697	0.3372	0.3716	0	0.0031	0.0006	0.0003	0.0002	0.0389	0.0665	0.0918	0.0558	0.2222	0.1	0.2227	0.2803	0.0633	0	0		
	2015	0	0	0	0	0	0	0.002	0.0006	0.0007	0.0001	0	0.0001	0	0	0	0	0	0	0	0.2228	0.1	0.2227	0.2803	0.0633	0	0		
	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.4999	0.0803	0.1679	0.1433	0.2268	0	0		

Source: Author's own calculations.

Table 6. Results of testing the null hypothesis for up and down waves for DJIA

Item	The wave beginning	The wave end	Direction of price movement	C-C						O-O						O-C						Overnight					
				J-B		L		W		A-D		J-B		L		W		A-D		J-B		L		W		A-D	
1	18.06.1901	15.10.1903	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
2	15.10.1903	18.01.1906	Up	0	0.0001	0	0	0	0	0	0.0001	0	0	0	0.0001	0	0	0	0	0.0001	0	0	0	0.0001	0	0	
3	18.01.1906	21.11.1907	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
4	21.11.1907	05.11.1909	Up	0	0.0146	0.0004	0.0002	0.0001	0	0.0146	0.0004	0.0002	0.0001	0	0	0	0	0	0	0.0146	0.0004	0.0002	0.0001	0	0		
5	05.11.1909	24.12.1914	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
6	24.12.1914	03.11.1919	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
7	03.11.1919	25.08.1921	Down	0	0.0366	0.0054	0.0044	0.0014	0	0.0366	0.0054	0.0044	0.0014	0	0	0	0	0	0.0366	0.0054	0.0044	0.0014	0	0	0		
8	25.08.1921	29.08.1929	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
9	29.08.1929	09.07.1932	Down	0	0.0001	0	0	0	0	0.0001	0	0	0	0	0	0	0	0	0.0001	0	0	0	0	0	0		
10	09.07.1932	10.03.1937	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
11	10.03.1937	29.04.1942	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
12	29.04.1942	14.12.1961	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
13	14.12.1961	26.06.1962	Down	0	0.0001	0	0	0	0	0.0004	0	0	0	0	0.0003	0	0	0	0	0.0003	0	0	0	0	0		
14	26.06.1962	11.02.1966	Up	0	0	0	0	0	0	0	0	0	0	0	0.0003	0	0	0	0.0003	0	0	0	0	0	0		
15	11.02.1966	11.10.1966	Down	0.8069	0.1	0.9254	0.9126	0.9216	0.8024	0.1	0.9568	0.9493	0.9551	0.7843	0.1	0.9115	0.8974	0.926	0.7843	0.1	0.9115	0.8974	0.926	0	0		
16	11.10.1966	06.12.1968	Up	0	0.1	0.4066	0.4751	0.1408	0.0016	0.1	0.6493	0.7777	0.3418	0.0017	0.1	0.7348	0.8286	0.4437	0.0017	0.1	0.7348	0.8286	0.4437	0	0		
17	06.12.1968	26.05.1970	Down	0	0.0697	0.0495	0.0379	0.017	0.0012	0.0857	0.0948	0.0845	0.03	0.0017	0.1	0.1143	0.101	0.0464	0.0017	0.1	0.1143	0.101	0.0464	0	0		
18	26.05.1970	12.01.1973	Up	0	0.0002	0	0	0	0	0.0006	0	0	0	0	0.0011	0.0001	0	0	0.0011	0.0001	0.0001	0	0	0	0		
19	12.01.1973	10.12.1974	Down	0.0003	0.0449	0.068	0.1309	0.0301	0.0044	0.1	0.4198	0.6594	0.3007	0.0022	0.0513	0.1715	0.365	0.0845	0.0022	0.0513	0.1715	0.365	0.0845	0	0		
20	10.12.1974	24.09.1976	Up	0.164	0.1	0.3175	0.4074	0.3102	0.1848	0.1	0.2754	0.3767	0.2114	0.0936	0.1	0.1432	0.1576	0.1202	0.0936	0.1	0.1432	0.1576	0.1202	0	0		
21	24.09.1976	02.03.1978	Down	0.4173	0.0826	0.1705	0.1607	0.2233	0.8806	0.1	0.1582	0.1402	0.1797	0.4098	0.0923	0.1062	0.089	0.1247	0.4098	0.0923	0.1062	0.089	0.1247	0	0		
22	02.03.1978	25.08.1987	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
23	25.08.1987	20.10.1987	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
24	20.10.1987	20.01.2000	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
25	20.01.2000	11.10.2002	Down	0	0.0083	0.0002	0.0001	0	0	0.0175	0.0002	0.0001	0	0	0.0034	0.0001	0	0	0.0034	0.0001	0.0001	0	0	0	0		
26	11.10.2002	12.10.2007	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
27	12.10.2007	10.03.2009	Down	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
28	10.03.2009	31.03.2017	Up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		

Source: Author's own calculations.