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NEGOTIATION OFFER EVALUATION WITH MIDIA: INTEGRATING ASPIRATION AND RESERVATION LEVELS INTO SCORING SYSTEM

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Abstract

Multi-issue negotiations require evaluating offers across multiple conflicting criteria, making decision-making complex and sensitive to negotiator's preferences. This paper aims to develop and apply the MIDIA (Multi-Criteria Method Integrating Distances to Ideal and Anti-Ideal Points) method as a novel scoring approach for multi-issue negotiation support. MIDIA integrates aspiration and reservation levels as ideal and anti-ideal benchmarks, enabling a balanced evaluation of offers, and introduces a flexible parameter α to regulate the relative emphasis between aspiration-driven and reservation-driven orientations, accommodating diverse negotiation strategies. This dual-reference-point framework is behaviorally informed, reflecting empirical evidence that decision-makers naturally consider both positive and negative aspects when evaluating alternatives. Grounded in the classical Hurwicz criterion, MIDIA provides a psychologically realistic and adaptable framework for negotiation analysis. Using a case study involving three negotiation issues: price, delivery time, and warranty, ten supplier offers were assessed under different weighting schemes. The results demonstrate how MIDIA captures both the stability and sensitivity of offers to changes in preference orientation and criterion weighting. Comparative analysis shows strong consistency between MIDIA ($\alpha = 0.5$) and the established TOPSIS method, while highlighting MIDIA's unique ability to flexibly balance aspiration and reservation perspectives. The findings confirm that MIDIA offers a transparent, adaptable, and behaviorally meaningful framework for negotiation support, with potential for further validation through empirical studies and systematic comparisons with other multi-criteria decision-making methods.

Keywords: multiple criteria decision making, reference points, negotiation scoring system, negotiation support.

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1 Introduction

Negotiation is a complex decision-making process that often involves multiple, sometimes conflicting, criteria (Bazerman and Moore, 2009; Thompson, 2005). Each party typically evaluates offers based on various aspects such as price, quality, and delivery time, making negotiations inherently multiple-criteria problems (Raiffa, Richardson and Metcalfe, 2002; Wachowicz and Roszkowska, 2021). In such multi-issue negotiations, negotiators often rely on structured tools to evaluate and compare offers.

One widely adopted approach is the use of scoring systems, which enable the quantitative assessment of alternatives based on predefined evaluation criteria and facilitate trade-offs among them. By translating qualitative and quantitative judgments into numerical scores, these systems help maintain consistency and objectivity throughout the negotiation process (Wachowicz, 2013).

A negotiation scoring system can be applied individually (asymmetric support) or mutually (symmetric support). Individually, it helps negotiators evaluate offers, plan concession strategies, track negotiation progress, and generate counteroffers, while analyzing reciprocity in concessions. Mutually, it supports both parties or a third-party mediator in suggesting fair solutions, maximizing joint contract value, verifying agreement efficiency, and identifying potential improvements. Accurate scoring is essential to ensure agreements that reflect true preferences (Wachowicz, 2013).

Moreover, scoring systems can be integrated into Negotiation Support Systems (NSS) to enhance trade-off generation, counteroffer construction, and overall negotiation analysis, thereby facilitating transparent and effective decision-making throughout the negotiation process. The studies (Kersten and Lai, 2007; Wachowicz, 2010a) provide a historical overview of negotiation support systems (NSS) and e-negotiation systems (ENS), discussing their models, architectures, and applications in practice, research, and training. It also examines research results and theoretical frameworks related to computer-based support for negotiation processes.

A wide range of Multiple Criteria Decision Making (MCDM) methods is commonly applied in the construction of negotiation scoring systems. These include SMART (Simple Multi Attribute Rating Technique) (Kersten and Noronha, 1999), AHP (Analytic Hierarchy Process) (Mustajoki and Hämäläinen, 2000), TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution) (Roszkowska and Wachowicz, 2015; Wachowicz and Błaszczyk, 2013; Wachowicz and Roszkowska, 2025), ELECTRE TRI (ÉLimination Et Choix Traduisant la REalité) (Wachowicz, 2010b), MARS (Measuring Attractiveness near Reference Situations) (Górecka, Roszkowska and Wachowicz, 2016) and UTA (Jarke, Jelassi and Shakun, 1987; Wachowicz and Roszkowska, 2022), Hellwig's (Roszkowska et al., 2022), Flexible and Interactive Tradeoff (FITradeoff) multicriteria method (de Almeida et al., 2016; Frej, Morais and Almeida, 2022; Frei et al., 2024), Ordered Weighted Average (OWA) (Leão E Silva Filho and Costa Morais, 2019), VIP (Variable Interdependent Parameters) Analysis (Clímaco and Dias, 2006). It should be emphasized that this list represents only a fraction of the broader spectrum of MCDM approaches. The methods are highlighted here to illustrate the diversity of techniques employed in negotiation analysis, ranging from classical scoring and ranking models to behaviorally oriented, reference-point-based frameworks. Furthermore, many of these approaches have been extended in the literature to address imperfect or incomplete information, as well as fuzzy or uncertain preference structures, thereby expanding their applicability in negotiation contexts.

Among the wide range of MCDM approaches, reference point-based methods deserve particular attention, as they explicitly incorporate aspiration levels (ideal outcomes) and reservation levels (minimally acceptable outcomes). By anchoring preferences around these benchmarks, they provide a realistic and flexible basis for negotiation analysis (Thompson, 2005; Wachowicz, Brzostowski and Roszkowska, 2012).

Traditional scoring methods, such as TOPSIS or VIKOR, offer structured ways to rank alternatives but often rely on a single reference point or fixed decision-maker priorities, which limits their behavioral realism. To address these shortcomings, this paper introduces the MIDIA (Multi-Criteria Method Integrating Distances to Ideal and Anti-Ideal Points) method. MIDIA considers both aspiration and reservation levels simultaneously and applies a flexible parameter α to balance their influence, thereby reflecting diverse negotiation strategies.

The study addresses the following research question:

RQ: *How effectively can the MIDIA method evaluate and rank offers in multi-issue negotiations by incorporating both aspiration- and reservation-driven perspectives, and how do its results compare with those of established methods such as TOPSIS?*

Developed in earlier research (Roszkowska and Filipowicz-Chomko, 2024), MIDIA uses a weighted system to capture balance and asymmetry in assessing alternatives based on their distances from the ideal and anti-ideal points. The parameter α determines the relative importance of closeness to the ideal versus remoteness from the anti-ideal. When $\alpha = 1$, the method reduces to the Hellwig measure (Hellwig, 1968, 1981), confirming its role as a generalization of established techniques.

The MIDIA is grounded in the structure of human judgment, which naturally evaluates both positive and negative aspects of decision options (Ordóñez, Connolly and Coughlan, 2000), and builds on classical decision-making principles such as the Hurwicz criterion balancing optimism and pessimism (Gaspars-Wieloch, 2024, 2014; Hurwicz, 1951). It also integrates elements of TOPSIS and VIKOR, combining distance-based reasoning with compromise-oriented logic.

The key contribution of this paper lies in applying MIDIA to negotiation scoring, where both subjective preferences and reference points must be considered. The α parameter enables modeling of different strategic orientations, capturing whether a negotiator emphasizes reaching an ideal outcome or avoiding unacceptable ones. This approach introduces a flexible and psychologically grounded tool for analyzing negotiation offers.

The remainder of the paper is structured as follows. Section 2 provides a comparative overview of reference points and distance-based multiple criteria decision-making methods. Section 3 introduces the MIDIA method in detail. Section 4 presents an illustrative example in the context of negotiation scoring systems, demonstrating the application of MIDIA and discussing the results. Finally, Section 5 concludes the paper and outlines directions for future research.

2 Reference point-based and distance-based MCDM methods: A comparative perspective

Multiple Criteria Decision Making refers to a family of methods designed to support decision-making in contexts where multiple, often conflicting criteria must be considered simultaneously (Amor et al., 2023; Ferreira, Ilander and Ferreira, 2019; Trzaskalik, 2014). MCDM methods address various types of problems, including ranking alternatives, selecting the best option, classifying options into categories, and analyzing performance across multiple criteria. These approaches help support decisions involving trade-offs between conflicting objectives.

A particularly important subset of MCDM methods includes those based on the concept of reference points and distance-based evaluations (Hellwig, 1968, 1981; Hwang and Yoon, 1981; Konarzewska-Gubała, 1989; Nermend, 2023; Opricovic and Tzeng, 2004; Özçil and Aytaç Adali, 2025; Roszkowska, Filipowicz-Chomko and Wachowicz, 2020; Wachowicz, Brzostowski and Roszkowska, 2012). Reference points serve as benchmarks for evaluating and comparing alternatives. These points can be categorized as internal, derived from the performance data of the alternatives under consideration and computed directly from the dataset, or external, defined subjectively by decision-makers or based on expert knowledge (Özçil and Aytaç Adali, 2025; Roszkowska, Filipowicz-Chomko and Wachowicz, 2020). The most common examples of internal reference points include the ideal point, which aggregates the best values for all criteria (e.g., highest benefits and lowest costs), and the anti-ideal point, which represents the worst values (lowest benefits and highest costs) (Hwang and Yoon, 1981; Opricovic and Tzeng, 2004). These points help define the boundaries of performance and guide compromise-based ranking or selection. In contrast, external reference points are not tied to the actual performance of alternatives but reflect predefined standards, target profiles, or hypothetical ideal scenarios introduced by decision-makers (Özçil and Aytaç Adali, 2025).

These points enable greater flexibility, especially in cases where decision-makers have prior expectations or when comparisons need to be made against policy goals or expert-defined thresholds. The distinction between internal and external reference points is fundamental in shaping how preferences are modeled and how decisions are supported in MCDM frameworks.

In MCDM techniques, distance measures play a crucial role in evaluating the closeness of alternatives to reference points such as ideal or anti-ideal solutions (Ciardiello and Genovese, 2023; Shih, Shyur and Lee, 2007; Shyur and Shih, 2024). Among these, the Euclidean distance (the L_2 Minkowski metric) is the most commonly used due to its simplicity, symmetry, and geometric interpretability. It calculates the straight-line distance between two points in a multi-dimensional space and serves as a fundamental component in classical methods such as TOPSIS (Hwang and Yoon, 1981) and its many variants.

The Euclidean distance is a special case of the more general Minkowski L_p metrics in n-dimensional space. For instance, when $p = 1$, the metric becomes the Manhattan (or city block) distance, and for $p = \infty$, it becomes the Chebyshev distance (Shih, Shyur and Lee, 2007). The Manhattan distance calculates the sum of the absolute differences across criteria and is less sensitive to outliers (Ciardiello and Genovese, 2023; Shih, Shyur and Lee, 2007). In contrast, the Chebyshev distance focuses on the maximum deviation across criteria and is often used in compromise models, particularly where the worst-case performance is of critical importance. It calculates the maximum absolute difference between corresponding attributes and is suitable for scenarios emphasizing extreme variations (Shyur and Shih, 2024). Beyond these classical norms, alternative distance measures within the Minkowski family, such as the Mahalanobis distance, are also employed (Chang et al., 2010; Shih, Shyur and Lee, 2007). The Mahalanobis distance accounts for correlations between criteria and is especially useful when criteria are interdependent. It has been applied in modified versions of TOPSIS (Liu et al., 2019; Ponce and Alcaraz, 2013) and Hellwig's method (Roszkowska, 2024). Ultimately, the choice of distance metric significantly influences the ranking of alternatives and should be aligned with the nature of the decision problem, the structure of the data, and the preferences of decision-makers.

We now present a brief review of selected multiple criteria methods based on reference points (see Table 1).

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), originally proposed by Hwang and Yoon in 1981 (Hwang and Yoon, 1981), is one of the most frequently applied methods across various disciplines (Behzadian et al., 2012; Zavadskas et al., 2016), widely used in both theoretical studies and real-world applications (Çelikbilek and Tüysüz, 2020; Palczewski and Sałabun, 2019b; Pandey, Komal and Dincer, 2023; Salih et al., 2019). This reference point-based approach determines the optimal alternative as the one closest to the ideal solution while being farthest from the anti-ideal solution. To evaluate alternatives, their Euclidean distances to the ideal solution, called the separation measure S^{+}, and to the negative-ideal solution, called the separation measure S^{-}, are calculated. The relative closeness of each alternative to the ideal solution, denoted as T_i, is then calculated using the formula $T_i = \frac{S^{-}}{S^{-} + S^{+}}$. This relative closeness serves as the criterion for ranking the alternatives. TOPSIS is widely preferred by decision-makers for its ease of use, clear logic, and minimal computational requirements, making it accessible even without advanced mathematical knowledge. Despite its strengths, the method does not account for behavioral or psychological factors in decision-making. Over time, many modifications and hybrid models have been proposed, incorporating various types of data, methodological enhancements, and the method's ongoing development has been thoroughly documented in recent literature (Behzadian et al., 2012; Palczewski and Sałabun, 2019b; Zavadskas et al., 2016; Zyoud and Fuchs-Hanusch, 2017).

VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) is a multiple criteria decision-making method focused on finding a compromise solution when facing conflicting criteria (Opricovic and Tzeng, 2004). Unlike methods such as TOPSIS, which use distance to the ideal solution, VIKOR is based on compromise programming and introduces two main indicators: the utility measure S_i reflecting overall performance across all criteria and the regret measure R_i indicating the worst performance in any criterion. These are combined into a compromise index Q_i using a parameter α that balances group satisfaction and individual regret. A typical choice, $\alpha = 0.5$, ensures equal weight to both. Due to its ability to propose a solution that is "closest to the ideal" and "acceptable to the group," VIKOR is often favored in multi-stakeholder environments or when trade-offs between collective and individual preferences must be transparently managed (Yazdani and Graeml, 2014).

Hellwig's method, introduced in 1968, also known as the Taxonomic Measure of Development (Hellwig, 1968), is a reference point-based approach used to evaluate alternatives based on their distance from the ideal solution. While it shares conceptual similarities with the TOPSIS procedure, the original formulation of Hellwig's method focuses solely on the distance from the ideal point.

A notable extension of this method was proposed by Hellwig in 1981 (Hellwig, 1981). This variant incorporates both the ideal and anti-ideal points by evaluating the relative closeness of each alternative to the ideal point in comparison to the distance between the ideal and anti-ideal points. By introducing the concept of relative closeness, the method becomes more robust and aligns more closely with other multiple criteria decision-making techniques. This variant of Hellwig's method can be viewed as a special case of more general frameworks, such as MIDIA (Roszkowska and Filipowicz-Chomko, 2024). Over time, the Hellwig's method has been refined and adapted to a wide range of problems and applications, with comprehensive reviews provided in the literature (Roszkowska, 2024).

The BIPOLAR method, proposed by Konarzewska-Gubała (1989), is an outranking technique designed for sorting and ranking problems based on a synthesizing preference relational system. Unlike TOPSIS or Hellwig, it uses two sets of reference alternatives: one with desirable (good) solutions and another with non-acceptable (bad) ones. It compares each alternative to these sets using outranking relations, resulting in a partial ranking where some alternatives may be considered indifferent. The method has been modified (Trzaskalik, 2021, 2023; Trzaskalik, Sitarz and Dominiak, 2019) and applied in various fields (Górecka, 2017, 2020; Tłuczak, 2018).

The MIDIA (Multi-Criteria Method Integrating Distances to Ideal and Anti-Ideal Points) method, originally proposed by Roszkowska and Filipowicz--Chomko (2024), is a decision-making technique that extends classical distance-based approaches by simultaneously incorporating two reference points: the ideal and anti-ideal solutions. MIDIA accounts for decision-makers' preferences through a tunable parameter α, which adjusts the emphasis on proximity to the ideal versus remoteness from the anti-ideal, enabling both optimistic and pessimistic evaluations within a single framework. Compared to TOPSIS, which balances closeness to ideal and distance from the anti-ideal using a fixed ratio, and VIKOR, which introduces compromise rankings via utility and regret, MIDIA offers greater flexibility by allowing asymmetric weighting of these reference points, thereby better capturing behavioral preferences in decision-making.

EDAS (The Evaluation based on Distance from Average Solution) is a multi-criteria decision-making method developed by Keshavarz Ghorabaee et al. (2015). Unlike methods based on ideal solutions, EDAS evaluates alternatives relative to the average value of each criterion. It uses two measures: Positive Distance from Average and Negative Distance from Average, depending on whether the criterion is beneficial or non-beneficial. EDAS is computationally efficient and offers intuitive interpretation by comparing alternatives to the average rather than to an ideal. Comparative studies with methods such as VIKOR, TOPSIS, SAW (Simple Additive Weighing), and COPRAS (COmplex PRoportional ASsessment) (Keshavarz Ghorabaee et al., 2015) have shown that EDAS provides stable and consistent results across different weighting scenarios.

MABAC (Multi-Attributive Border Approximation Area Comparison) is a multi-criteria method initially proposed by Pamučar and Ćirović (2015). The method establishes a border approximation area (BAA) and assesses the position of each alternative relative to this boundary. The best alternative is selected based on the distance between each alternative and the BAA. Pamučar and Ćirović (2015) demonstrated that MABAC delivers more stable and consistent results compared to SAW, COPRAS, TOPSIS, MOORA (The Multi-Objective Optimization Ratio Analysis), and VIKOR.

The Best-Worst Method (BWM), developed by Jafar Rezaei in 2015 (Rezaei, 2015), is a multi-criteria decision-making approach that derives optimal weights for decision criteria through pairwise comparisons. It relies on two reference points: the most important (Best) and the least important (Worst) criteria, which act as anchors in the evaluation. Other criteria are compared only to these reference points, reducing the number of comparisons and enhancing consistency. BWM uses a distance-based optimization model that minimizes the maximum absolute deviation between the derived weight ratios and the decision-maker's preferences, typically employing the Chebyshev norm to ensure reliable and consistent results. The BWM technique differs from the rest in that it is not directly distance-based but provides a foundation for other MCDM methods by determining accurate weights through optimization based on pairwise comparisons between the best and worst criteria.

Wachowicz, Brzostowski and Roszkowska (2012) analyze the practical usefulness of several reference-point-based methods, such as TOPSIS, VIKOR, Hellwig's method, and BIPOLAR, in the framework of negotiation analysis in the construction of offer evaluation systems. Their paper explores their potential to support decision-makers in dynamic and context-dependent negotiation scenarios. Numerous modifications to the TOPSIS and Hellwig's methods were proposed to better tailor these techniques to the situational context of negotiations, accounting for factors such as linguistic preferences (Piasecki and Roszkowska, 2018), appropriate distance measures in TOPSIS (Wachowicz and Błaszczyk, 2013; Wachowicz and Brzostowski, 2012), dealing with uncertainty (Roszkowska, 2021; Roszkowska and Wachowicz, 2015), or evaluation of offers outside the negotiation space (Wachowicz and Roszkowska, 2025), among others. These enhancements aim to increase the flexibility and relevance of the methods in real-world negotiation settings.

Table 1 provides a comparative overview of selected reference point- and distance-based MCDM methods, highlighting their differences in reference point types, distance metrics, and evaluation approaches.

Table 1: MCDM methods by reference point type and distance measure

Method	References	Reference point	Distance metric	Description
TOPSIS	Hwang and Yoon (1981); Shih, Shyur and Lee (2007); Liu et al. (2019)	Ideal & Anti-Ideal	Euclidean, Manhattan, Chebyshev, Mahalanobis	Ranks alternatives by closeness to the ideal and distance from the anti-ideal
VIKOR	Opricovic and Tzeng (2004); Yazdani and Graeml (2014)	Ideal	Manhattan, Compromise Index	Finds a compromise solution between group utility and individual regret
Hellwig	Hellwig (1968, 1981); Roszkowska (2021)	Ideal (and Anti-Ideal in variant)	Euclidean, Mahalanobis	Distance to ideal point; variant includes anti-ideal for relative closeness
MIDIA	Roszkowska and Filipowicz-Chomko (2024)	Ideal & Anti-Ideal	Euclidean, adaptable to other norms	Integrates ideal and anti-ideal levels via α −weighted closeness and remoteness from reference points
BIPOLAR	Konarzewska--Gubała (1989); Trzaskalik (2021); Górecka (2020)	Good & Bad Reference Sets	Outranking relations (qualitative distance)	Uses desirable and undesirable reference sets for sorting
EDAS	Keshavarz Ghorabaee et al. (2015)	Average Solution	Absolute deviation from average (positive/negative distance)	Assesses deviation from average rather than ideal
MABAC	Pamučar and Ćirović (2015)	Border Approximation Area (BAA)	Distance from BAA	Uses BAA as a benchmark to compare alternatives
BWM	Rezaei (2015)	Best and Worst Criteria	Chebyshev for weight optimization	Derives weights through pairwise comparisons with reference criteria

In summary, the reviewed MCDM methods differ in their use of reference points, distance metrics, and approaches to ranking or evaluating alternatives. Classical methods like TOPSIS and Hellwig's measure rely on ideal and anti-ideal points, while VIKOR emphasizes compromise between utility and regret. Other approaches, such as EDAS and MABAC, use alternative benchmarks like averages or border approximation areas, and BWM focuses on deriving robust criteria weights. MIDIA uniquely combines ideal and anti-ideal reference points with a tunable $\boldsymbol{\alpha}$ parameter, enabling flexible consideration of both aspiration- and reservation-driven preferences. This comparative perspective highlights the strengths and limitations of existing methods and positions MIDIA as a novel, behaviorally informed approach for multi-issue negotiation support. In the following section, we detail the MIDIA methodology and demonstrate its applicability through a case study.

3 A multi-criteria method integrating distances to ideal and anti-ideal points

3.1 Algorithm of the MIDIA method

The following presents the fundamental steps of the MIDIA method proposed in the study by Roszkowska and Filipowicz-Chomko (2024). The method aims to comprehensively evaluate alternatives based on their distances from the ideal and anti-ideal solutions, incorporating the decision-maker's preferences through a weighting parameter.

Assume a set of m alternatives $A = \{A_1, A_2, \dots, A_m\}$ and n criteria $C = \{C_1, C_2, \dots, C_n\}$. For benefit criteria, higher values are preferred; for cost criteria, lower values are better. Let x_{ij} be the value of the alternative A_i for criterion C_j $(i = 1,2,\dots,m;\ j = 1,2,\dots,n)$.

Step 1: Construct the decision matrix:

$$D = [x_{ij}] \tag{1}$$

where $i = 1,2,\dots,m;\ j = 1,2,\dots,n.$

Step 2: Define the weight vector:

$$w = [w_1, \dots, w_n] \tag{2}$$

where $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1,\ j = 1, \dots, n.$

Many methods exist for determining weights in multi-criteria decision-making. Weights reflect the importance of each criterion and ensure that more relevant ones have a greater impact on the final result. A comprehensive review of weighting approaches is provided by da Silva et al. (2021).

Step 3a: Determine the ideal solution:

$$I = [x_1^+, \dots, x_n^+] \tag{3}$$

where: $x_j^+ = \max_i x_{ij}$ for the benefit criterion;

$x_j^+ = \min_i x_{ij}$ for the cost criterion $(j = 1, \dots, n)$.

Step 3b: Determine the anti-ideal solution:

$$AI = [x_1^-, \dots, x_n^-] \tag{4}$$

where: $x_j^- = \min_i x_{ij}$ for the benefit criterion;
$x_j^- = \max_i x_{ij}$ for the cost criterion ($j = 1, \dots, n$).

Step 4: Normalize the decision matrix:

$$\bar{D} = [\bar{x}_{ij}] \tag{5}$$

using the normalization formula:

$$\bar{x}_{ij} = \begin{cases} \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{for the benefit criterion} \\ \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} & \text{for the cost criterion} \end{cases} \tag{6}$$

where: $i = 1, \dots, m, j = 1, \dots, n$.

Although the original MIDIA approach employed standardization, this study adopts the min-max normalization method, as it is more suitable for addressing the negotiation problem. It should be noted that other normalization techniques, such as vector or sum normalization, are also widely used (Milani et al., 2005; Palczewski and Sałabun, 2019a).

Step 5. Construct the weighted normalized matrix:

$$\tilde{D} = [\tilde{x}_{ij}] \tag{7}$$

where $\tilde{x}_{ij} = w_j \bar{x}_{ij}$ ($i = 1,2,\dots,m$; $j = 1,2,\dots,n$).

Step 6a. Calculate the distances of i-th alternative A_i from the ideal I by using the Euclidean distance measure:

$$dI_i(A_i, I) = \mathrm{E}(\tilde{A}_i, \tilde{I}) = \sqrt{\sum_{j=1}^{n} \left(\tilde{x}_{ij} - \tilde{x}_j^+\right)^2} \tag{8}$$

where $\tilde{A}_i = [\tilde{x}_{i1}, \dots, \tilde{x}_{in}]$, $\tilde{I} = [\tilde{x}_1^+, \dots, \tilde{x}_n^{\mp}]$, $\tilde{x}_{ij}, \tilde{x}_j^+$ are weighted normalized values x_{ij} and x_j^+, respectively.

Step 6b. Calculate the distances of i-th alternative A_i from the anti-ideal AI by using the Euclidean distance measure:

$$dAI_i(A_i, AI) = \mathrm{E}(\tilde{A}_i, \widetilde{AI}) = \sqrt{\sum_{j=1}^{n} \left(\tilde{x}_{ij} - \tilde{x}_j^-\right)^2} \tag{9}$$

where $\tilde{A}_i = [\tilde{x}_{i1}, \dots, \tilde{x}_{in}]$, $\widetilde{AI} = [\tilde{x}_1^-, \dots, \tilde{x}_n^{\mp-}]$, $\tilde{x}_{ij}, \tilde{x}_j^-$ are weighted normalized values x_{ij} and x_j^-, respectively.

Other distance measures can be used in multi-criteria methods (Shih, Shyur and Lee, 2007; Wang and Wang, 2014).

Step 7a: Compute closeness to ideal:

$$DI_i = 1 - \frac{dI_i}{d^{+-}} \tag{10}$$

where $d^{+-} = \sqrt{\sum_{j=1}^{n}\left(\tilde{x}_j^{+} - \tilde{x}_j^{-}\right)^2}$ is the distance between the weighted normalized ideal and anti-ideal solutions.

Step 7b: Compute closeness to anti-ideal:

$$DAI_i = \frac{dAI_i}{d^{+-}} \tag{11}$$

where $d^{+-} = \sqrt{\sum_{j=1}^{n}\left(\tilde{x}_j^{+} - \tilde{x}_j^{-}\right)^2}$ is the distance between the weighted normalized ideal and anti-ideal solutions.

Let us observe that, when using the min-max normalization formula, the distance d^{+-} simplifies to $d^{+-} = \sqrt{\sum_{j=1}^{n} w_j{}^2}$. This results from the fact that, after normalization, the ideal point takes the value 1 for all criteria, and the anti-ideal point takes the value 0, making the squared differences equal to the squared weights.

Step 8: Calculate the aggregated measure:

$$D(\alpha)_i = \alpha\, DI_i + (1-\alpha) DAI_i \tag{12}$$

The parameter $\alpha \in [0,1]$ represents the weight of the distance to the ideal solution. The parameter $\alpha = 1$ yields an ideal-focused strategy, $\alpha = 0$, an anti-ideal-focused one, while $\alpha = 0.5$ balances both perspectives.

Step 9: Rank alternatives:
Sort $D(\alpha)_{\mathrm{i}}$ in descending order. Higher values indicate better alternatives.

3.2 Theoretical basis and motivation for using the MIDIA method in scoring systems

Before empirical validation, it is important to outline the theoretical basis of the MIDIA method, providing a coherent framework for its potential applications. The primary motivation for considering MIDIA in scoring system design lies in its unique ability to evaluate negotiation offers by integrating both aspiration-oriented (ideal) and reservation-oriented (anti-ideal) perspectives. This dual-reference structure aligns well with the requirements of scoring systems, which must balance the pursuit of desirable outcomes against the avoidance of unacceptable ones.

Using a weighting parameter (α), the method allows negotiators to model their evaluation criteria according to specific goals. Emphasizing proximity to the ideal point reflects an aspiration-driven approach aimed at maximizing value, while focusing on distance from the anti-ideal point supports a reservation-

-driven approach that avoids unacceptable outcomes. The α parameter acts as a balance between these two perspectives: $\alpha = 1$ ($D(1) = DI$) reflects a purely aspiration-based evaluation, $\alpha = 0$ ($D(0) = DAI$) emphasizes avoidance of poor outcomes, and intermediate values represent a balanced trade-off, reflecting real-world negotiations where both goals are relevant.

The MIDIA system, like other negotiation scoring systems, can be applied in both asymmetric and symmetric modes. In asymmetric mode, it supports individual negotiators in evaluating offer profitability, planning concession strategies, and monitoring negotiation dynamics. In symmetric mode, MIDIA allows both parties or a third-party mediator to suggest fair solutions, maximize joint contract value, and resolve potential deadlocks. This dual-use capability ensures that MIDIA is versatile and applicable throughout the negotiation process, from pre-negotiation planning to final agreement assessment, while remaining consistent with the principles of scoring systems.

Given these theoretical properties, the MIDIA method provides a strong foundation for developing scoring systems that are both adaptable and grounded in decision theory. This motivates further exploration and practical implementation, which will be presented in the next section.

4 Evaluating offers using MIDIA in a multi-issue negotiation

This section presents a practical example of applying the MIDIA method to evaluate offers in a multi-issue negotiation. It demonstrates how MIDIA supports decision-making when multiple criteria are involved and facilitates effective comparison of competing offers.

4.1 Case description

A manufacturing company is negotiating a contract with a supplier for electronic components. The negotiation involves three key issues:

- Unit Price (lower is better),
- Delivery Time (days) (shorter is better),
- Warranty Period (months) (longer is better).

The company's procurement team has clearly defined aspiration levels, the most desired outcome for each issue, and reservation levels, the minimum acceptable values for each issue. Those reference points are presented in Table 2.

Table 2: Aspiration and reservation levels for negotiation issues

Criterion	Aspiration level	Reservation level
Unit Price (€)	5	11
Delivery Time (days)	1	7
Warranty Period (months)	24	12

The ten supplier offers in Table 3 were selected to represent a range of realistic negotiation outcomes, allowing us to illustrate how MIDIA evaluates different scenarios. By applying varying α values, the example highlights both the method's strengths, flexibility, and ability to identify robust offers, and its limitations, such as sensitivity to shifts in preference orientation and the need for careful weight elicitation.

Table 3: Data for the example

Offer	Unit Price	Delivery Time	Warranty Period
O1	6	7	12
O2	5	4	18
O3	10	3	12
O4	7	7	12
O5	9	1	18
O6	7	2	24
O7	11	1	24
O8	9	3	18
O9	6	7	24
O10	8	5	24

In the assessment, three different weighting systems, reflecting the relative importance of each issue, are applied, as presented in Table 4. System 1 uses equal weights for all criteria, serving as a benchmark for balanced decision--making. In Systems 2 and 3, price is treated as the most important criterion, reflecting its typical dominance in procurement decisions. The remaining two criteria, delivery time and warranty, together balance the influence of price, but their relative importance differs. System 2 gives more weight to delivery, while System 3, to warranty. This setup allows us to demonstrate how MIDIA responds when the key criterion remains constant, while the relative importance of the other two criteria changes. Naturally, many other weighting combinations could be considered, but these three systems serve as clear and illustrative examples of the method's sensitivity to varying priorities without overcomplicating the results section.

Table 4: The system weights

Weighs	Unit Price	Delivery Time	Warranty Period
System 1: Equal weights (Case 1)	0.(3)	0.(3)	0.(3)
System 2: Non-equal weights (Case 2)	0.5	0.3	0.2
System 3: Non-equal weights (Case 3)	0.5	0.2	0.3

4.2 Offer evaluation using the MIDIA approach

The company uses MIDIA to evaluate offers by balancing closeness to the aspiration and reservation levels. All steps of the MIDIA procedure are carried out sequentially, starting from constructing the decision matrix (see Table 3) and defining the weight vector (see Table 4).

The ideal and anti-ideal points based on max and min values (see Formulas (3)-(4)) are: $I = [5,1,24]$, and $AI = [11,7,12]$. Note that these points correspond to the aspiration and reservation levels defined in Table 2. After normalization and weighting (see Formulas (5)-(7)), the distances to the ideal and anti-ideal solutions are calculated using Euclidean metrics (Formulas (8)-(9)). These distances are then used to compute the DI and DAI measures (Formulas (10)-(11)). Finally, the aggregated $D(\alpha)$ values are determined for $\alpha = 0, 0.1, \ldots, 1$ using Formula (12).

The indicator $D(\alpha)$ aggregates offer values by balancing preferences between ideal and anti-ideal solutions. The parameter $\alpha \in [0,1]$ indicates the evaluation focus:

- α = 1: Ideal-oriented (aspiration-driven) – focus on closeness to the aspiration offer,
- α = 0: Anti-ideal-oriented (reservation-driven) – focus on staying as far away as possible from the reservation offer,
- α = 0.5: Balanced – equal weight to both ideal and anti-ideal points.

Tables 5-7 present the computed values of $D(\alpha)$ for ten offers assessed across weighting coefficients $\alpha \in [0, 1]$, using three different weighting systems.

Table 5: The values of measure $D(\alpha)$ for different coefficients α (Case 1)

Offer	**$D(\alpha)$ Values for Different Coefficients α**											**Max-Min**
	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	
O1	0.844	0.838	0.832	0.827	0.821	0.815	0.809	0.803	0.797	0.791	0.785	0.060
O2	0.816	0.777	0.738	0.698	0.659	0.620	0.580	0.541	0.501	0.462	0.423	0.394
O3	0.752	0.718	0.684	0.650	0.617	0.583	0.549	0.516	0.482	0.448	0.415	0.337
O4	0.707	0.696	0.684	0.673	0.661	0.649	0.638	0.626	0.615	0.603	0.592	0.115
O5	0.674	0.658	0.643	0.627	0.612	0.596	0.581	0.565	0.550	0.534	0.519	0.155
O6	0.674	0.658	0.643	0.627	0.612	0.596	0.581	0.565	0.550	0.534	0.519	0.155
O7	0.518	0.515	0.511	0.507	0.504	0.500	0.496	0.493	0.489	0.485	0.482	0.036
O8	0.481	0.451	0.420	0.390	0.360	0.329	0.299	0.269	0.239	0.208	0.178	0.303
O9	0.397	0.379	0.362	0.345	0.328	0.310	0.293	0.276	0.259	0.241	0.224	0.173
O10	0.385	0.363	0.340	0.318	0.295	0.273	0.251	0.228	0.206	0.184	0.161	0.224
max	0.844	0.838	0.832	0.827	0.821	0.815	0.809	0.803	0.797	0.791	0.785	
min	0.385	0.363	0.340	0.318	0.295	0.273	0.251	0.228	0.206	0.184	0.161	
diff	0.459	0.476	0.492	0.509	0.525	0.542	0.558	0.574	0.591	0.607	0.624	

Table 6: The values of measure $D(\alpha)$ for different coefficients α (Case 2)

Offer	**$D(\alpha)$ Values for Different Coefficients α**											**Max-Min**
	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	
O1	0.750	0.747	0.743	0.740	0.737	0.734	0.731	0.727	0.724	0.721	0.718	0.032
O2	0.585	0.545	0.506	0.466	0.426	0.387	0.347	0.308	0.268	0.228	0.189	0.396
O3	0.750	0.724	0.699	0.673	0.648	0.622	0.597	0.571	0.546	0.520	0.495	0.255
O4	0.862	0.847	0.831	0.816	0.800	0.785	0.769	0.754	0.738	0.723	0.708	0.155
O5	0.544	0.538	0.531	0.525	0.519	0.512	0.506	0.500	0.493	0.487	0.481	0.063
O6	0.580	0.565	0.551	0.537	0.522	0.508	0.493	0.479	0.464	0.450	0.435	0.144
O7	0.452	0.448	0.444	0.440	0.436	0.433	0.429	0.425	0.421	0.417	0.413	0.040
O8	0.676	0.648	0.621	0.593	0.565	0.538	0.510	0.483	0.455	0.427	0.400	0.276
O9	0.351	0.340	0.328	0.316	0.304	0.292	0.280	0.268	0.257	0.245	0.233	0.119
O10	0.541	0.522	0.504	0.485	0.467	0.448	0.430	0.411	0.393	0.374	0.356	0.185
max	0.862	0.847	0.831	0.816	0.800	0.785	0.769	0.754	0.738	0.723	0.718	
min	0.351	0.340	0.328	0.316	0.304	0.292	0.280	0.268	0.257	0.228	0.189	
diff	0.511	0.507	0.504	0.500	0.496	0.493	0.489	0.485	0.482	0.495	0.529	

Table 7: The values of measure $D(\alpha)$ for different coefficients α (Case 3)

Offer	**$D(\alpha)$ Values for Different Coefficients α**											**Max-Min**
	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	
O1	0.776	0.771	0.766	0.761	0.755	0.750	0.745	0.740	0.735	0.729	0.724	0.052
O2	0.585	0.545	0.506	0.466	0.426	0.387	0.347	0.308	0.268	0.228	0.189	0.396
O3	0.833	0.814	0.796	0.778	0.759	0.741	0.722	0.704	0.685	0.667	0.649	0.184
O4	0.862	0.847	0.831	0.816	0.800	0.785	0.769	0.754	0.738	0.723	0.708	0.155
O5	0.643	0.632	0.622	0.612	0.602	0.592	0.581	0.571	0.561	0.551	0.540	0.102
O6	0.487	0.479	0.471	0.463	0.455	0.447	0.439	0.431	0.423	0.415	0.407	0.080
O7	0.423	0.421	0.418	0.415	0.413	0.410	0.408	0.405	0.402	0.400	0.397	0.026
O8	0.676	0.648	0.621	0.593	0.565	0.538	0.510	0.483	0.455	0.427	0.400	0.276
O9	0.255	0.246	0.236	0.227	0.217	0.208	0.198	0.189	0.179	0.170	0.160	0.095
O10	0.541	0.522	0.504	0.485	0.467	0.448	0.430	0.411	0.393	0.374	0.356	0.185
max	0.862	0.847	0.831	0.816	0.800	0.785	0.769	0.754	0.738	0.729	0.724	
min	0.255	0.246	0.236	0.227	0.217	0.208	0.198	0.189	0.179	0.170	0.160	
diff	0.607	0.601	0.595	0.589	0.583	0.577	0.571	0.565	0.559	0.560	0.564	

4.3 Interpretation of case studies

This section presents the interpretation of the results obtained using the MIDIA method applied to the evaluated offers. The analysis uses aggregated evaluation values $D(\alpha)$, computed for $\alpha = 0$ to 1 in steps of 0.1, under three weighting systems reflecting different priorities for price, delivery time, and warranty. The aim is to illustrate how the evaluation of each offer changes as the emphasis gradually moves from a reservation-driven orientation ($\alpha = 0$) to an aspiration--driven one ($\alpha = 1$). By observing the behavior of $D(\alpha)$ across the full range of α, and comparing results under different weighting systems, we gain insights into two key aspects:

(1) the sensitivity of offers to shifts in preference orientation (aspiration vs. reservation), and
(2) the influence of changes in the relative importance of evaluation criteria.

Tables 5 to 7 contain the numerical results for each offer under the three weighting systems. Offers with flat or stable $D(\alpha)$ profiles can be considered robust, as their evaluation remains consistent regardless of the value of α. Offers with more fluctuation in their scores are more dependent on the decision-maker's preference orientation or on how heavily each criterion is weighted in the analysis.

Figures 1 to 3 provide a visual representation of how the evaluation scores of the ten offers evolve with changing values of α. These figures help to identify possible shifts in rankings and further illustrate the influence of preference orientation and criterion weighting on the final assessment.

Case 1: *Equal importance of price, delivery time, and warranty*
This scenario assumes a balanced decision-making strategy where all three criteria (price, delivery time, and warranty) are treated with equal importance.

The "diff" row (max – min at each α) illustrates the spread between the best and worst performing offers, effectively capturing the sensitivity of the decision landscape to changes in α (see Table 5). The spread increases steadily from 0.459 at $\alpha = 0$ to a peak of 0.624 at $\alpha = 1$, suggesting that as decision-makers adopt a more aspiration-oriented perspective, the differences between offers become more pronounced and the selection of a top offer becomes more consequential.

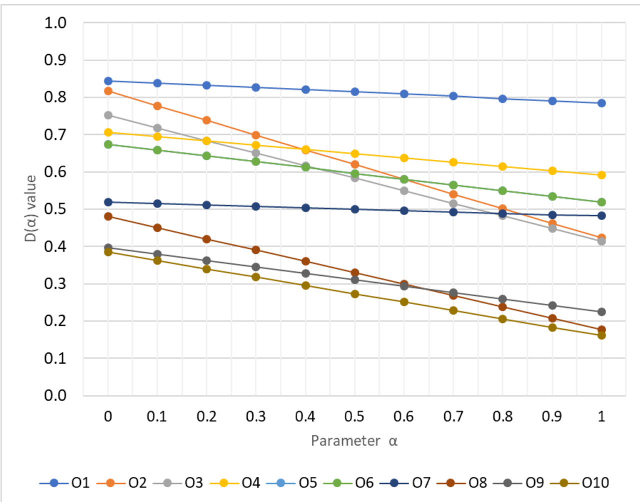

Figure 1: $D(\alpha)$ values across offers for Case 1

A general trend observed across offers is that $D(\alpha)$ decreases monotonically as α increases (see Table 5, Figure 1) and preference evaluation changes its focus from a reservation-driven orientation to an aspiration-driven one. However, the rate of decline varies significantly across offers, highlighting differing sensitivities to changes.

Offer O1 stands out for its remarkable consistency. With an initial $D(\alpha)$ value of 0.844 at $\alpha = 0$ and a final value of 0.785 at $\alpha = 1$, the total drop is just 0.060, the smallest among all offers. This variation makes O1 a highly stable and reliable option regardless of the negotiator's attitude toward aspiration. Offer O2 begins with a strong value (0.816 at $\alpha = 0$) but undergoes a sharp decline, falling to 0.423 by $\alpha = 1$. The total spread of 0.394 suggests that O2 is highly favorable under reservation-driven orientation but becomes less competitive as preference shifts toward aspiration outcomes. Offer O4 demonstrates strong stability, with a range of only 0.115 across all α levels. This consistent performance makes O4 a suitable candidate for decision-makers who are undecided or prefer a compromise between aspiration and reservation perspectives. Among the evaluated offers, O5 and O6 exhibit identical $D(\alpha)$ values across all coefficients α. This means their evaluation scores remain constant regardless of the preference weighting between aspiration and reservation levels. Offer O7 is noteworthy for its low variability, which drops by just 0.036. While it remains near the middle or lower end of the ranking, its resistance to fluctuation suggests that these options provide predictable performance regardless of α. However, their overall attractiveness is moderate to low. Offers O9 and O10 consistently receive the lowest scores across all α levels, indicating weak performance both in avoiding poor outcomes and in striving for ideal scenarios.

The Spearman correlation coefficients between the rankings at different values of α are generally very high, especially for values of α close to each other, indicating strong consistency in offer rankings when preference weighting shifts slightly (see Table 8). Correlations gradually decrease as the difference between α values increases, with the lowest values (0.704) observed between the extreme $D(0)$ and $D(1)$. This suggests that while rankings remain relatively stable overall, more pronounced changes in the aspiration-reservation emphasis can lead to moderate shifts in the relative order of offers.

Table 8: The Spearman correlation coefficient for $D(\alpha)$ and different coefficients α (Case 1)

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	1.000	1.000									
0.2	1.000	1.000	1.000								
0.3	0.988	0.988	0.988	1.000							
0.4	0.964	0.964	0.964	0.988	1.000						
0.5	0.904	0.904	0.904	0.952	0.964	1.000					
0.6	0.823	0.823	0.823	0.870	0.905	0.965	1.000				
0.7	0.811	0.811	0.811	0.858	0.894	0.953	0.988	1.000			
0.8	0.763	0.763	0.763	0.823	0.858	0.942	0.977	0.988	1.000		
0.9	0.704	0.704	0.704	0.763	0.811	0.895	0.954	0.965	0.988	1.000	
1.0	0.704	0.704	0.704	0.763	0.811	0.895	0.954	0.965	0.988	1.000	1.000

Case 2: *Weighting: price 0.5, delivery 0.2, warranty 0.3*
This setup strongly favors price, making it the dominant evaluation criterion. As a result, the ranking of offers differs noticeably from Case 1.

As in Case 1, the "diff" row (Max – Min at each α level) highlights the spread between the best and worst performing offers, reflecting the sensitivity of the decision environment to changes in the coefficient α. In Case 2, this spread starts at 0.511 for $\alpha = 0$, dips slightly to 0.482 at $\alpha = 0.8$, and then climbs to a peak of 0.529 at $\alpha = 1.0$. Compared to Case 1, where the spread increased more steadily and substantially (from 0.459 to 0.624), Case 2 suggests a slightly more stable decision landscape, although still influenced by shifts in preference orientation.

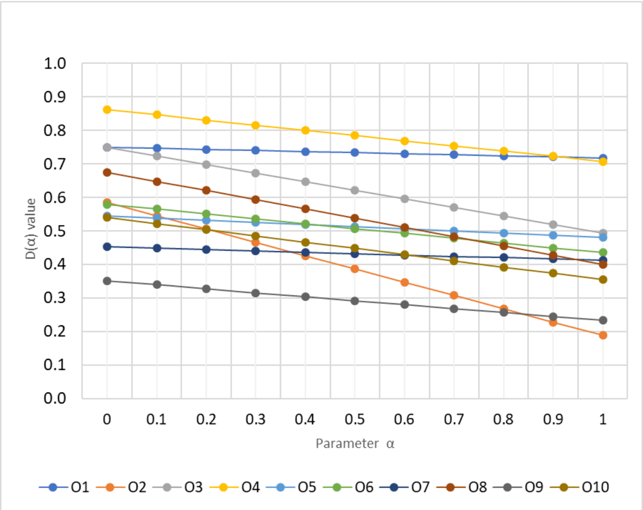

Figure 2: $D(\alpha)$ values across offers for Case 2

A general pattern remains consistent with Case 1 (see Table 6, Figure 2), where $D(\alpha)$ decreases monotonically as α increases, reflecting the transition in decision-maker's focus from reservation-based to aspiration-based evaluation. However, as in Case 1, the rate of decline varies across offers, indicating differing sensitivities to changes in α.

Offer O1 is the most stable in Case 2, with $D(\alpha)$ values declining very gradually from 0.750 at $\alpha = 0$ to 0.718 at $\alpha = 1.0$, with a minimal drop of only 0.032. Its low variability suggests robustness across decision-maker's orientations. Offer O2, in contrast, shows a sharp and consistent drop from 0.585 to 0.189, resulting in the largest range (0.396). Similar to Case 1, O2 performs relatively well under a reservation-driven orientation but loses appeal rapidly as more weight is placed on aspiration orientation, making it a less attractive choice for ambitious negotiators. Offer O4 displays a moderate and smooth decline from 0.862 to 0.708, with a spread of 0.155. Though this is a larger drop than its Case 1 equivalent (which had a range of only 0.115), O4 still maintains above-average scores across all α values, making it a strong candidate for those seeking a balanced compromise. Offer O5 and O6 exhibit moderate variability with spreads of 0.063 and 0.144, respectively. While their changes are not as minimal as in Case 1 (where evaluation offers remained completely constant), they still demonstrate relatively low sensitivity, which may be appealing in uncertain decision environments. Offer O7 is again notable for its low fluctuation, decreasing from 0.452 to 0.413 (a difference of only 0.040). This is consistent with the behavior of stable middle-performing offers in Case 1. While not top performers, their consistency may offer value in predictable outcomes. Offers O8 and O10 exhibit steeper declines, with $D(\alpha)$ ranges of 0.276 and 0.185, respectively. These offers tend to perform reasonably well under reservation-driven preferences but degrade noticeably under aspiration-oriented views. Offers O9 and O10, similar to Case 1, occupy the lower end of the $D(\alpha)$ scale across all α levels, suggesting limited effectiveness under both reservation and aspiration perspectives.

Case 2 shows a similar decreasing trend in Spearman correlation coefficients with increasing α as observed in Case 1, indicating that rankings shift as preference weighting changes (see Table 9). However, the decline in stability is more noticeable in Case 2, with the lowest value (0.667) observed between $D(0)$ and $D(1)$, compared to 0.704 in Case 1. This means that rankings in Case 2 are more sensitive to changes in α.

Table 9: The Spearman correlation coefficient for $D(\alpha)$ and different coefficients α (Case 2)

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	0.984	1.000									
0.2	0.960	0.988	1.000								
0.3	0.925	0.964	0.988	1.000							
0.4	0.878	0.927	0.964	0.988	1.000						
0.5	0.867	0.903	0.952	0.976	0.988	1.000					
0.6	0.867	0.903	0.952	0.976	0.988	1.000	1.000				
0.7	0.820	0.855	0.915	0.927	0.952	0.976	0.976	1.000			
0.8	0.796	0.842	0.903	0.915	0.939	0.952	0.952	0.988	1.000		
0.9	0.738	0.794	0.867	0.891	0.927	0.939	0.939	0.976	0.988	1.000	
1.0	0.667	0.721	0.794	0.818	0.867	0.879	0.879	0.939	0.964	0.976	1.000

Case 3: *Weighting: price 0.5, delivery 0.3, warranty 0.2*
This case prioritizes cost while giving more weight to warranty than to delivery.

As in the previous cases, the "diff" row in Table 7 captures the spread between the highest and lowest-performing offers across the α. In Case 3, the spread starts at 0.607 for $\alpha = 0$ and remains relatively stable, fluctuating slightly (0.559 for $\alpha = 0.8$) before reaching 0.564 at $\alpha = 1.0$. This pattern is different from Case 1, where the spread increased steadily, and from Case 2, where it remained relatively narrow and peaked slightly at the end.

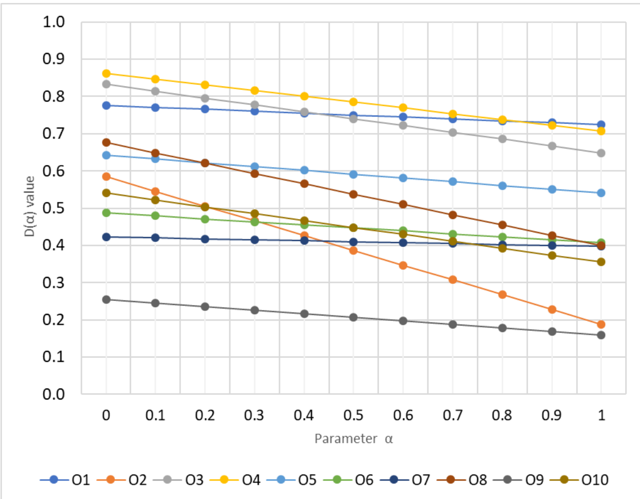

Figure 3: $D(\alpha)$ values across offers for Case 3

As with previous cases, most offers display a monotonic decrease in $D(\alpha)$ as α increases, reflecting the shift in decision-maker's focus from reservation-based to aspiration-oriented preferences. However, the extent of decline continues to vary between offers, providing insight into their robustness to changes in preference orientation. Offer O1 demonstrates strong stability, starting at 0.776 and ending at 0.724, with a total drop of only 0.052. While slightly less stable than in Case 2 (0.032) and slightly more than in Case 1 (0.060), O1 continues to stand out as a reliable offer across all decision attitudes. Offer O2, as in the previous cases, exhibits a sharp decline from 0.585 to 0.189, with a range of 0.396, identical to Case 2 and similar to its behavior in Case 1. This repeated pattern suggests that O2 performs best in reservation-focused contexts and rapidly loses competitiveness under aspiration-driven evaluations. Offer O3 starts high at 0.833 and drops to 0.649, with a range of 0.184. While less stable than O1, it maintains relatively strong performance and can be regarded as moderately good. Offer O4 again shows consistent and strong performance, dropping from 0.862 to 0.708 (range 0.155). This confirms the previous conclusion from Cases 1 and 2 that O4 is a versatile and balanced offer for negotiators seeking compromise. Offer O5 is more sensitive in Case 3 than in Case 2, with a spread of 0.102 (compared to just 0.063 before). It is still reasonably stable but shows some increased variation in this scenario. Offer O6 drops from 0.487 to 0.407, with a moderate spread of 0.080, suggesting slightly higher stability compared to its evaluation in Case 2 (0.144), but lower starting performance. Offer O7 is again the most stable offer with a minimal change of only 0.026, reinforcing its status as a predictable but mid-to-low performer, regardless of the aspiration level. Offer O8 continues to show a significant drop (0.276), suggesting sensitivity to aspiration-level changes and declining utility. Offer O9 is the weakest performer overall, with values ranging from 0.255 to 0.160, and a relatively low spread of 0.095, indicating consistent underperformance across all α levels. Offer O10, with a drop of 0.185, exhibits similar performance to previous cases, where it takes middle position under reservation perspective, but its competitiveness decreases as α increases.

Case 3 shows greater ranking stability across α values than both Case 1 and Case 2 do, with a Spearman correlation of 0.794 for $D(0)$ and $D(1)$ rankings and compared to 0.704 and 0.667, respectively. This means that Case 3 rankings are less sensitive to changes in preference weighting, which makes them more consistent.

Table 10: The Spearman correlation coefficient for $D(\alpha)$ and different coefficients α (Case 3)

Spearman Coefficient	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.000										
0.1	1.000	1.000									
0.2	0.988	0.988	1.000								
0.3	0.976	0.976	0.988	1.000							
0.4	0.952	0.952	0.964	0.988	1.000						
0.5	0.903	0.903	0.915	0.952	0.976	1.000					
0.6	0.891	0.891	0.903	0.927	0.964	0.988	1.000				
0.7	0.891	0.891	0.903	0.927	0.964	0.988	1.000	1.000			
0.8	0.867	0.867	0.879	0.891	0.927	0.964	0.988	0.988	1.000		
0.9	0.842	0.842	0.855	0.867	0.903	0.952	0.976	0.976	0.988	1.000	
1.0	0.794	0.794	0.818	0.830	0.879	0.927	0.964	0.964	0.976	0.988	1.000

Summing up, in Case 1, the spread shows a steady increase (0.459 for $\alpha = 0 \rightarrow 0.624$ for $\alpha = 1$), with O1 and O7 as the most stable offers, O2 and O8 being the most sensitive. Offer O1 is the best, while O9 and O10 are consistently the poorest performers. The overall trend suggests that rising aspiration sharpens distinctions. In Case 2, the spread remains narrow and stable (0.511 for $\alpha = 0 \rightarrow 0.529$ for $\alpha = 1$), with O1 and O7 again as the top stable offers, O2, O3, and O8 as the most sensitive. Offer O4 performs the best, while O9 and O10 remain weak, with the overall trend showing moderate differentiation in rankings. In Case 3, the spread is broad but steady (0.607 for $\alpha = 0 \rightarrow 0.564$ for $\alpha = 1$), O1 and O7 are still the most stable, O2 and O8 are the most sensitive. Offer O4 represents the top performance, while O10 continues to exhibit the lowest performance and uniform divergence, defining the overall pattern.

4.4 Discussion

The analysis of the three cases demonstrates the versatility and sensitivity of the MIDIA method in multi-criteria decision-making contexts, highlighting its ability to capture two critical aspects: the sensitivity of offers to changes in preference orientation (α) and the influence of varying weights for criteria such as price, delivery, and warranty. Variations in these weights significantly affect the ranking of alternatives, emphasizing the importance of aligning them with the decision-maker's true preferences. The parameter α plays a crucial role by balancing the distance to the anti-ideal and ideal solutions, thereby reflecting different decision attitudes from reservation-driven to aspiration-driven orientations.

The MIDIA-based rankings are robust for small changes in α, meaning that slight changes in the decision maker's emphasis on anti-ideal vs ideal points do not significantly affect the ranking of the offers. However, if the decision-

-making orientation shifts substantially (e.g., from $\alpha = 0.0$ to $\alpha = 1.0$), the rankings change more noticeably, which highlights the sensitivity of the scoring system to negotiator preferences. Some offers show robust performance across a wide range of α values and weighting schemes, indicating stability and making them reliable choices when preferences are uncertain. Across all three cases, Offer O1 consistently emerges as a stable and robust option, suitable for all negotiation attitudes. Offer O7 also maintains a strong and balanced profile. Offer O2, though initially attractive, shows significant sensitivity, making it risky under changing preference strategies. Case 3 introduces a scenario where overall differences between offers remain large but steady, suggesting a stable yet stratified decision landscape. This makes the selection of top offers highly consequential, even under small changes in α.

Finally, we conducted a comparison between the results generated by the MIDIA method and those produced by other well-established reference point-based methods, specifically TOPSIS. To better understand the patterns and relationships highlighted by these approaches, we analyzed and discussed the outcomes of MIDIA for parameter values $\alpha = 0$, $\alpha = 0.5$, and $\alpha = 1$, in parallel with the results obtained from TOPSIS (see Figures 4-6).

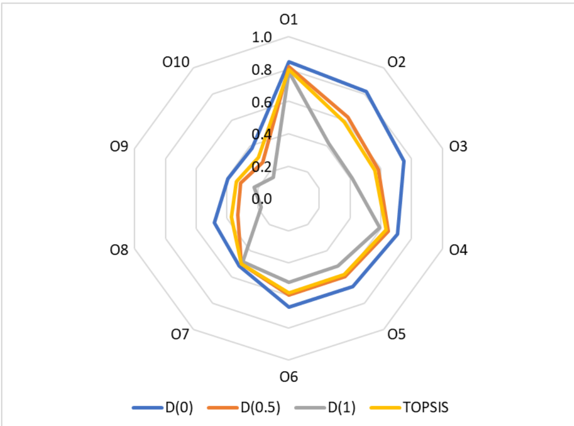

Figure 4: Graphical representation of values for TOPSIS and for $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ in MIDIA (Case 1)

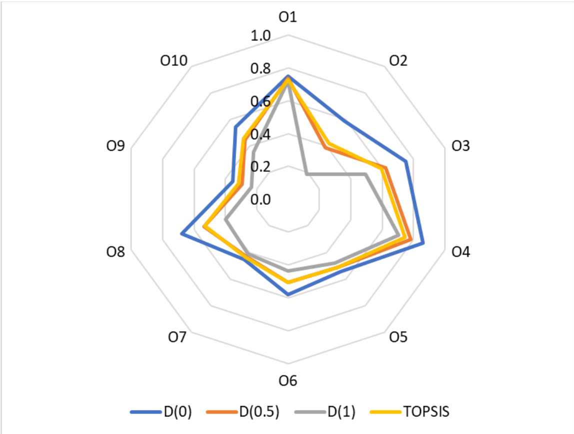

Figure 5: Graphical representation of values for TOPSIS and for $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ in MIDIA (Case 2)

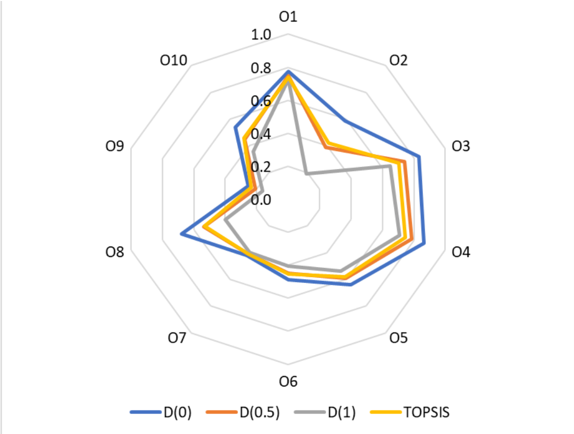

Figure 6: Graphical representation of ranking for TOPSIS and for $\alpha = 0$, $\alpha = 0.5$, $\alpha = 1$ in MIDIA (Case 3)

The results presented in Figures 4-6 indicate that the rankings derived from TOPSIS closely align with those obtained using MIDIA with $\boldsymbol{\alpha} = \mathbf{0.5}$, demonstrating a strong consistency between those two approaches. In Cases 1 and 2, identical rankings were observed for both the TOPSIS and $\boldsymbol{D}(\mathbf{0.5})$ methods. In Case 3, the rankings differed slightly, with only offers O3 and O7 switching positions: O7 ranked eighth and O3 ninth in the $\boldsymbol{D}(\mathbf{0.5})$ method, while the opposite occurred in TOPSIS. However, when comparing the rankings produced by the $\boldsymbol{D}(\mathbf{0})$, $\boldsymbol{D}(\mathbf{1})$, and TOPSIS methods, more significant discrepancies were observed.

The example demonstrates the applicability of the MIDIA method in unifying aspiration and reservation perspectives within a single framework, offering a robust tool for comprehensive evaluation. However, the results underline the critical role of careful weight elicitation for the implementation of this technique in practice.

Summing up, the results of the case study clearly show that the MIDIA method effectively ranks the ten supplier offers across the three negotiation issues, price, delivery time, and warranty, under varying weighting schemes. The method captures both the stability of robust offers and the sensitivity of others to changes in preference orientation (α) and criterion weights. Comparison with the established TOPSIS method confirms strong consistency for $\alpha = 0.5$, while also highlighting MIDIA's distinctive capacity to balance aspiration and reservation perspectives. These findings directly address the research question posed in the Introduction, demonstrating that MIDIA provides a robust, adaptable, and behaviorally meaningful framework for multi-issue negotiation support.

Finally, the analysis demonstrates the key strengths and limitations of the MIDIA method. Its advantages include flexibility in reflecting both aspiration- and reservation-driven perspectives, robustness of rankings under small changes in α, transparency in evaluating trade-offs among multiple negotiation issues, and the ability to identify offers that remain stable across varying weighting schemes. At the same time, some limitations and practical challenges should be noted: outcomes are sensitive to large shifts in preference orientation (α), careful elicitation of criteria weights is essential, and the illustrative cases provide demonstration rather than full empirical validation, meaning further studies are needed to assess performance in diverse real-world settings. Overall, these observations provide a balanced evaluation of MIDIA's applicability, highlighting its versatility while acknowledging areas requiring careful consideration.

5 Conclusions

This study demonstrated the application of the MIDIA method as an innovative scoring approach tailored for multi-issue negotiations where offers must be evaluated across conflicting dimensions. By incorporating aspiration and reservation levels as the ideal and anti-ideal reference points, MIDIA enables a nuanced assessment of negotiation offers that reflects both the desire to achieve favorable outcomes and the need to avoid unacceptable ones. The flexibility introduced by the α parameter allows negotiators to adjust the relative weight of these perspectives, supporting different strategic orientations from optimistic to cautious decision-making. The illustrative cases confirmed the method's capacity to capture the dynamic interplay between criteria weights and decision-maker preferences, providing a transparent and behaviorally realistic framework for offer evaluation in negotiation settings.

Nonetheless, it is important to acknowledge the limitations of this study. The examples provided serve primarily as illustrative demonstrations rather than comprehensive validations of MIDIA's performance. Future research should focus on extensive empirical analyses and simulation studies involving diverse datasets, varied α values, and alternative weighting schemes to rigorously evaluate the robustness and practical applicability of the method. Additionally, initial comparisons of rankings obtained through the $D(0.5)$ with TOPSIS suggest promising consistency; however, more comprehensive and systematic comparative studies with other methods, such as VIKOR, SAW, or COPRAS, are also needed. Exploring different normalization procedures, choosing reference points (internal or external), and distance metrics could further enhance the model's adaptability and accuracy. Such deeper investigations are essential to establish MIDIA as a reliable tool in negotiation support systems and broader multi-criteria decision-making contexts.

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