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Milena Bieniek*
Jarosław Banaś**

The Multi-Criteria Nature of Classical Decisions in Wholesale-Price Contracts with Product Exchange

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Abstract

This theoretical article investigates the link between multi-criteria decision-making and wholesale-price contracts with consumer product exchange. It also presents optimal solutions to those contracts within a game-theoretic framework. In the proposed model, the retailer manages exchanges and shares associated costs with consumers. A Stackelberg game is used to determine optimal pricing and ordering strategies, with the manufacturer as leader and the retailer as follower. The supply chain is modeled analytically under stochastic multiplicative demand. Because of mathematical complexity, optimal quantities are derived numerically in Mathematica based on analytic profit functions.

Keywords: multiple criteria decision making, wholesale-price contract, Stackelberg game, product exchange.

1 Introduction

The classical wholesale-price contract is among the simplest and most widely used supply chain contracts. Under this scheme, the retailer pays the manufacturer a fixed, quantity-independent price for each unit purchased. These

* Maria Curie-Sklodowska University, Faculty of Management, Poland,
e-mail: milena.bieniek@umcs.lublin.pl, ORCID: 0000-0001-9686-7650.

** Maria Curie-Sklodowska University, Faculty of Management, Poland,
e-mail: jaroslaw.banas@mail.umcs.pl, ORCID: 0000-0002-5684-6828.

contracts are two-tier topology contracts and although they are common in practice, such contracts typically fail to coordinate the supply chain effectively. Their popularity stems largely from their simplicity and typical structure under the newsvendor framework. In most cases, the upstream agent can satisfy all downstream agent orders and acts as a Stackelberg leader, setting the wholesale price. Then the retailer, as follower, chooses the retail price and order quantity. In push contracts, the manufacturer secures revenue upfront, while the retailer bears demand risk (Cachon, 2003; Cachon and Lariviere, 2005; Hezarkhani and Kubiak, 2010).

Supply chain contracts inherently involve multi-criteria decision-making (MCDM), since multiple, often conflicting objectives must be balanced. For example: efficiency vs. responsiveness: Larger inventories improve responsiveness but raise costs and reduce efficiency, or facility cost vs. delivery speed: Fewer distribution centers lower costs but lengthen delivery times (Ravindran, ed., 2016). Wholesale-price contracts also involve conflicting objectives between the parties. The manufacturer typically seeks to maximize profit by setting the wholesale price as high as possible, while the retailer aims to protect its margin by negotiating a lower wholesale price and optimizing the retail price and order quantity. This tension leads to the classical double marginalization effect, where the total profit of the decentralized supply chain is lower than that of a centralized system.

Beyond profit maximization, other criteria could exist that shape decisions. Manufacturers consider production efficiency, stable capacity utilization, market share expansion, and risk transfer. Retailers focus on inventory factors, demand satisfaction, inventory risk, and consumer loyalty. In contracts with product exchange, retailers prioritize minimizing losses from returned and replaced products, while manufacturers value revenue stability and production planning reliability. Each contract decision variable therefore serves multiple purposes. Wholesale price is not just a transfer parameter, but a mechanism for balancing efficiency and fairness between the parties. Retail price is a lever influencing both short-term revenue and long-term positioning. Order quantity is a compromise between retailer risk exposure and manufacturer cost efficiency. Exchange-related terms (e.g., buy-back prices, return rates) are explicit coordination tools. Designing wholesale-price contracts thus constitutes a multi-objective optimization problem, requiring careful trade-offs between partners' objectives while improving overall efficiency (Tsay et al., 1999).

This paper analyzes the management of wholesale-price contracts with product exchange to identify decision rules that lead to optimal outcomes for both parties. Optimality is defined in game-theoretic terms, as an equilibrium in a Stackelberg game with the manufacturer as leader (Cachon and Lariviere, 2005). The supply chain under analysis consists of three entities: the manufacturer, the retailer, and the consumer. The retailer faces a classical newsvendor problem. Before the end of the selling season, the retailer orders inventory from the manufacturer at the wholesale price and then sells the products to consumers at retail prices within the same season. In this framework, non-defective items purchased by consumers may be exchanged for new ones, creating a reverse channel managed by the retailer. Any surplus inventory can be salvaged without capacity constraints. Consumer demand is stochastic and depends multiplicatively on the retail price.

The model builds on relationships among supply chain members introduced in Liu et al. (2020) and further expanded in Bieniek (2025) and Bieniek and Szapiro (2024). Liu et al. (2020) analyze wholesale transactions with price-free stochastic demand, treating the retail price as an exogenous variable. Bieniek (2025) refines this framework by introducing endogenous pricing and considering additive demand, while examining wholesale-price contracts under manufacturer-managed product exchanges. Bieniek and Szapiro (2024) extend the analysis by proposing four revenue-sharing contract scenarios under additive demand and optimizing their performance.

In this paper, we focus on the case of multiplicative demand within a wholesale-price contract. Specifically, we formalize and optimize the contract under the assumption that the retailer is responsible for managing product exchanges. In the numerical example, we evaluate the supply chain performance for this contract, with particular attention to the variability of exchange-related costs. The contributions of this paper are as follows:

1. Development of an analytical model of a wholesale-price contract with product exchange under multiplicative stochastic demand.

2. Optimization of the wholesale-price contract with exchange, including analytical expressions for the optimal retail price and order quantity, as well as a numerical solution for the optimal wholesale price.

3. Sensitivity analysis of optimal quantities and supply chain profits with respect to the change of return-related costs.

In the analysis of a wholesale price contract within a Stackelberg game framework studied here, elements typical of MCDM are presented in the decision structure. Alternatives refer to different combinations of decision variables: wholesale price, retail price, and order quantity. Attributes represent measurable outcomes, such as the profits of the manufacturer and the retailer. The decision space encompasses all feasible combinations of the triplet of decision variables subject to economic constraints. Although the problem is not a classical MCDM problem, the Stackelberg contract analysis includes MCDM elements, such as conflicting criteria (e.g. profit maximization for both the manufacturer and the retailer), and differing preferences of the participants. Thus, while the problem is formulated as a continuous optimization, its structure reflects aspects of MCDM in the context of the Stackelberg contract.

2 Literature review

In this section we list the most important papers on MCDM and wholesale-price contracts.

We begin with a review of the literature on MCDM. Multi-criteria decision making provides structured tools for evaluating alternatives based on multiple qualitative and quantitative criteria. In general, MCDM supports decisions involving conflicting criteria, and solutions are typically compromise-based (Aruldoss et al., 2013). Numerous MCDM methods can be classified in different ways depending on their complexity, weighting approaches, and handling of uncertainty. The most often used are as follows. AHP deals with a pairwise comparison of hierarchical criteria considering difference information (Saaty, 1987). FUZZY AHP is AHP with the fuzzy evaluation of the alternatives (van Laarhoven and Pedrycz, 1983). DEA is defined as a performance assessment of a set of homogeneous decision making units with multiple inputs and outputs (Charnes et al., 1978). TOPSIS makes evaluation based on the distance of an alternative to the ideal solution (Tzeng and Huang, 2011). ELECTRE outranks the relationship of the alternatives using pairwise comparison (Benayoun et al., 1966). PROMETHEE is an outranking method, such as ELECTRE, including several iterations (Brans, 1982). VIKOR is a compensatory version of TOPSIS that is based on minimizing the distance to the ideal solution using a linear normalization approach (Opricovic, 1998). SAW involves addition of scores representing the goal achievements considering all criteria that is multiplied by the criteria weights (Churchman and Ackoff, 1954). BMW identifies the best and the worst criteria followed by conducting a pairwise comparisons between each of the best and worst criteria and other ones (Rezaei, 2015).

The literature on MCDM methods is vast; the most recent and most cited items include, among others: Kumar and Pamucar (2025) with systematic review for the last two decades, Abdel-Basset et al. (2025) applying a multi-criteria decision-making approach to evaluate industry 5.0 technologies, and Oubrahim and Sefiani (2025) introducing integrated multi-criteria decision-making approach to evaluate sustainable supply chain performance.

We now list the most recent papers on wholesale-price contracts with stochastic demand.

In Zhao and Dou (2024), the authors integrate corporate social responsibility (CSR) and channel coordination in a supply chain under random yield and random demand. The supplier, facing random yield, determines the wholesale price and production input, while the producer decides on the order quantity and CSR investment. Nash equilibrium solutions for decentralized supply chains under wholesale-price and revenue-sharing contracts are analyzed. The authors propose a combined revenue- and CSR-sharing contract that achieves channel coordination and generates win-win outcomes for supply chain members and related stakeholders.

In Jammernegg et al. (2025), the authors propose a supply-chain objective maximization approach to coordinate a supplier-buyer relationship where both parties may act as behavioral decision makers. They present a general condition for the existence of a coordinating wholesale price based on a monotonicity assumption, which holds for a broad class of behavioral factors.

In Liu et al. (2025), three models are developed based on consumer choice behavior: a benchmark model without trade-in, a trade-in model with manufacturer entrustment, and an extended model with a wholesale-price contract. The equilibrium solutions of these models are derived and compared. The results show that introducing a wholesale-price contract can benefit the manufacturer, increase profits, and encourage trade-in, thereby achieving a win-win situation.

In Zheng et al. (2024), the authors analyze how wholesale-price discrimination affects different parties in a supply chain where a supplier distributes products through two competing retailers. They consider the case in which contract terms between the supplier and each retailer are unobservable to the rival retailer. Contract unobservability induces the supplier to set lower discretionary wholesale prices, which can outweigh the benefits of pricing flexibility. Interestingly, the lack of commitment benefits both retailers, improves supply chain efficiency, and increases consumer surplus.

In Li et al. (2025), the authors study contract strategies in a non-exclusive supply chain, where two manufacturers supply competing products to two retailers. The retailers may choose either revenue-sharing or wholesale-price contracts. The analysis highlights how contract arrangements are shaped by non-exclusive selling formats, product competition, and retailer competition. The findings show that intense retailer competition prevents either contract type from dominating. Negotiations over revenue-sharing ratios may fail, leaving wholesale-price contracts as the only viable option. However, wholesale-price contracts achieve win-win outcomes only under conditions of weak retailer competition and strong product competition.

Chen et al. (2025) extend the standard newsvendor model to a dynamic setting with price-dependent stochastic demand. Their model incorporates the retailer's fairness concerns. Using deterministic demand as a benchmark, they develop three static and dynamic models under bounded rationality, accounting for both multiplicative and additive demand. The results reveal that greater fairness concerns lead to higher retailer pricing but lower order quantities. Moreover, multiplicative stochastic disturbances have a stronger impact on deterministic demand than additive disturbances.

In Zhou et al. (2025), the authors construct a price competition model in which sellers consider both profits and peer-comparison outcomes. Two sellers offering substitutable products each set prices ex ante to maximize their expected utility, defined as the sum of profit and peer-comparison payoff. The analysis shows that peer comparison intensifies price competition, reducing sellers' profits and utilities, benefiting consumers, and lowering platform profits.

In Akhtar et al. (2025), the optimality of a low-carbon supply chain system is investigated. In the supply chain considered, the manufacturer and retailer operate under a wholesale pricecost sharing contract. The model accounts for variable demand rates and includes coordination mechanisms. The manufacturer covers part of the retailer's promotional costs, while the retailer supports the manufacturer's quality improvement and emission reduction efforts. A Stackelberg game framework is used to derive the system's optimality conditions. Numerical examples illustrate the solutions and show that coordination contracts provide higher benefits for both parties compared to non-coordination scenarios.

Taken together, these studies provide rich insights into wholesale-price contracts under various assumptions, including behavioral preferences, CSR, trade-in strategies, and discriminatory pricing. However, none considers wholesale-price contracts under multiplicative demand when consumer product exchanges are managed by the retailer. This paper addresses this gap. Specifically, we extend the models of Liu et al. (2020) and Bieniek (2025) by analyzing contracts with consumer exchanges under multiplicative demand.

3 Problem formulation

Let us consider a supply chain comprising a manufacturer and a retailer who make independent decisions. Within this arrangement, the manufacturer introduces a product, which is then channeled through the retailer. The retailer purchases the product at the wholesale price from a manufacturer, while consumers pay the retail price directly to the retailer. The manufacturer faces a unit cost for producing the item. The manufacturer offers a contract to the retailer, who may accept or reject it. Once the retailer accepts, the manufacturer-retailer interaction is modeled as a Stackelberg game where the manufacturer acts as the leader (von Stackelberg, 2011). Within this supply chain, the retailer allows consumers to exchange non-defective products. Returned products are subsequently inspected or repackaged before being resold as new items, which is the retailer's responsibility. The consumer's cost of handling returns encompasses reverse shipping fees, travel and time costs, and is further influenced by the responsiveness and convenience of the returns process. Table 1 outlines the parameters, assumptions, and decision variables of the model.

The scheme of product flow in the forward channel can be presented as:

$$\text{Manufacturer} \rightarrow \text{Retailer} \rightarrow \text{Consumer}$$

and in the backward channel:

$$\text{Retailer} \leftarrow \text{Consumer}$$

In the analytical model considered here, it is assumed that customer demand for the product depends on the retail price and is stochastic. Specifically, for $\varepsilon \in [A, B]$ being a random variable with cdf F independent of retail price and continuously differentiable pdf f, the demand on products is denoted by:

$$D(p, \varepsilon) = d(p)\varepsilon$$

with:

$$d(p) = a(p + h_c)^{-b}$$

being a deterministic demand (Ru and Wang, 2010). Let us stress that the demand is multiplicative and depends on the cost that the customer has to bear for exchange. In our model the average consumer and retailer exchange handling costs are assumed to be the product of exchange handling cost per unit by the probability that the exchange happens.

Table 1: Model parameters, notation and assumptions

Decision variables	
p	retail price per unit
z	inventory factor
q	order quantity
w	wholesale price per unit
Π_c	expected profit in the centralized channel
Π_m, Π_r	manufacturer's and retailer's expected profit
$c > 0$	unit production cost
v	unit salvage value $v < c$
h_c, h	$= \alpha H_c, \alpha H > 0$, where $\alpha \in [0,1]$ is a probability of exchange and H_c, H are average consumer and retailer exchange handling costs
$a > 0, b > 2$	deterministic demand parameters
ε	random variable with $\mathrm{E}[\varepsilon] = 1$ and $\sigma^2[\varepsilon] < \infty$
$F(x),\ f(x)$	cumulative distribution function (cdf) and probability distribution function (pdf) of ε with support $[A, B]$, $0 \leq A < B$
IFR	class of distributions with increasing failure rate $g(x) = \frac{f(x)}{1-F(x)}$
IGFR	class of distributions for which generalized failure rate $u(x) = \frac{xf(x)}{1-F(x)}$ is increasing
$\Lambda(z) = \int_A^z (z-\varepsilon) f(\varepsilon) d\varepsilon$	where $z - \Lambda(z) = \mathrm{E}[\min\{z, \varepsilon\}]$ and $\frac{\mathrm{d}\Lambda(z)}{\mathrm{d}z} = F(z)$
$c + h \leq w + h \leq p$	assures the existence of solution.

Based on the schemes we will outline profit equations for the centralized channel and the decentralized channel. In the centralized channel, the decision making is made by a central authority. Subsequently, we will present the expression for the expected profit in the centralized channel:

$$\Pi_c(p, q) = (p - v - h)\mathrm{E}[\min\{q, D(p, \varepsilon)\}] - (c - v)q$$

The anticipated profits of the decentralized channel can be represented by the following expressions. The manufacturer's expected profit is given by:

$$\Pi_m(w) = (w - c)q$$

and the retailer's expected profit is expressed as:

$$\Pi_r(p, q) = \Pi_c(p, q) - \Pi_m(w)$$

It is important to note that the expected profit of the retailer can be obtained by calculating the difference between the centralized expected profit and the manufacturer's expected profit.

Defining an inventory factor by:

$$z = q/(a(p + h_c)^{-b}) = q/(d(p))$$

we get:

$$\begin{aligned} \mathrm{E}[\min\{q, D(p, \varepsilon)\}] &= \mathrm{E}[\min\{zd(p), d(p)\varepsilon\}] = \\ &= d(p)\mathrm{E}[\min\{z, \varepsilon\}] = \\ &= d(p)(z - \Lambda(z)) \end{aligned}$$

(Petruzzi and Dada, 1999; Rubio-Herrero et al., 2015; Rubio-Herrero and Baykal-Gursoy, 2020) by the assumptions, which allows us to express the expected profit functions in newsvendor form.

4 Results

As a benchmark, we first analyze the centralized channel, where the decision maker selects order quantity and retail price to maximize system profit:

$$\max_{p,z\in[A,B]} \Pi_c(p, z) = d(p)\{(p - v - h)(z - \Lambda(z)) - (c - v)z\} \tag{1}$$

Theorem 4.1. *Under the assumptions, the problem defined by* (1) *has a unique solution* (p_c, z_c) *given by:*

$$p_c(z) = \frac{bh + h_c + bv}{b - 1} + \frac{bz(c - v)}{(b - 1)(z - \Lambda(z))} \tag{2}$$

and:

$$(p_c(z) - v - h)\bar{F}(z) - (c - v) = 0 \tag{3}$$

Proof. Note that $\frac{\mathrm{d}}{\mathrm{d}p}d(p) = -\frac{bd(p)}{p+h_c}$. We have:

$$\frac{\mathrm{d}\Pi_c(p, z)}{\mathrm{d}p} = \frac{d(p)}{p + h_c}\{(z - \Lambda(z))((1 - b)p + bh + h_c) + bzc\}$$

which by the first order condition $\frac{\mathrm{d}\Pi_c(p,z)}{\mathrm{d}p} = 0$ gives the formula for optimal price $p_c(z)$ defined by (2). The solution $p_c(z)$ is unique, since the gradient of the linear function of p which is equal to $(z - \Lambda(z))(1 - b)$ is negative which implies that the function is decreasing. Therefore, $\frac{\mathrm{d}\Pi_c(p,z)}{\mathrm{d}p} > 0$ for $p < p_c(z)$ and $\frac{\mathrm{d}\Pi_c(p,z)}{\mathrm{d}p} < 0$ for $p > p_c(z)$.

Now, let us note that:

$$\frac{\mathrm{d}\Pi_c(p,z)}{\mathrm{d}z} = \frac{\delta\Pi_c(p,z)}{\delta z} + \frac{\delta\Pi_c(p,z)}{\delta p}\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{\delta\Pi_c(p,z)}{\delta z}$$

Next, substituting formula (2) into formula (1) we get $\Pi_c(p_c(z), z) = \Pi_c(z)$, which is a continuous and smooth function. Then, the optimal decision z_c is given by the first order condition:

$$\frac{\mathrm{d}\Pi_c(z)}{\mathrm{d}z} = 0$$

which implies (3) and determines the ordering decision. Let us observe that:

$$\frac{\mathrm{d}\Pi_c(z)}{\mathrm{d}z} = \frac{d(p)}{(b-1)(z-\Lambda(z))}H(z) \tag{4}$$

where:

$$H(z) = (h_c + h)(z - \Lambda(z))\bar{F}(z) + bzc\bar{F}(z) - c$$

Hence, to prove the uniqueness of z_c it suffices to show that the function $H(z)$ has only one z-intercept. We get:

$$\mathrm{sgn}\frac{\mathrm{d}\Pi_c(z)}{\mathrm{d}z} \mid_{z=A} = \mathrm{sgn}H(z) \mid_{z=A} = \mathrm{sgn}(p - c - h) > 0$$

by assumptions and:

$$\mathrm{sgn}\frac{\mathrm{d}\Pi_c(z)}{\mathrm{d}z} \mid_{z=B} = \mathrm{sgn}H(z) \mid_{z=B} = \mathrm{sgn}(-c) < 0$$

Moreover:

$$H'(z) = \bar{F}(z)\{(h_c + h)(\bar{F}(z) - g(z)) + bc - bcu(z))\}$$

and:

$$H''(z)|_{H'(z)=0} = -f(z)H'(z) - \bar{F}(z)\{(h_c + h)(f(z) + g'(z)) + bcu'(z)\} < 0$$

by IGFR property (Barlow and Proschan, 1996). Therefore, $H(z)$ is unimodal, at first positive and then negative, and crosses the z-axes only once. Summarizing, since the first factor in (4) is positive, and $\Pi_c(z)$ increases at A and decreases at B, then there exists a unique maximum of $\Pi_c(z)$ with the unique optimal inventory factor $z = z_c$. The proof is complete.

□

In the decentralized channel, under the inverse induction method the retailer's problem:

$$\max_{p,z\in[A,B]} \Pi_r(p,z) = d(p)\{(p-v-h)(z-\Lambda(z))-(w-v)z\}$$

is solved first.

Then the manufacturer's problem:

$$\max_{w} \Pi_m(w) = d(p)(w-c)z$$

is considered.

The decisions to the decentralized problem formulated above are as follows.

Theorem 4.2. *Under the assumptions, the retailer's unique optimal price p_d is given by:*

$$p_d(z) = \frac{bv+h_c+bh}{b-1} + \frac{bz(w-v)}{(b-1)(z-\Lambda(z))}$$

and the retailer's optimal inventory factor $z = z_d$ is uniquely determined by:

$$(p_d(z)-v-h)\bar{F}(z) = w-v$$

Proof. The proof of the statements of the decentralized case is omitted as it is similar to the previous one in the case of a centralized channel.

□

Anticipating the retailer's best response, the manufacturer sets the wholesale price w to maximize his/her profit. This optimization can be reformulated in terms of the inventory factor z:

$$\max_{z\in[A,B]} \Pi_m(z)$$

can be obtained by expressing $p_d(z)$ and $w_d(z)$ in terms of z:

$$p_d(z) = \frac{h_c(z-\Lambda(z))+b(h+v)(zF(z)-\Lambda(z))}{(b-1)(z-\Lambda(z))-bz\bar{F}(z)}$$

and:

$$w_d(z) = (p_d(z)-v-h)\bar{F}(z)+v \tag{5}$$

We now need the following lemma.

Lemma 4.1. *The following results hold:*

1. *If $z \in [A, B]$ then the functions $p_d(z)$ and $w_d(z)$ have a single discontinuity point at z_0 defined by $(b-1)(z_0 - \Lambda(z_0)) - bz_0\bar{F}(z_0) = 0$.*

2. *The function $p(z) \geq 0$ if $z \in [z_0, B]$ and $p(z) < 0$ otherwise.*

3. *For $z \in [z_0, B]$ we get $p(z) \geq w(z) + h$.*

Proof. 1. Let $p_d(z) = \frac{L(z)}{M(z)}$ with:

$$M(z) = (b-1)(z - \Lambda(z)) - bz\bar{F}(z)$$

and:

$$L(z) = h_c(z - \Lambda(z)) + b(h+v)(zF(z) - \Lambda(z))$$

Let z_0 be the solution to $M(z) = 0$ in the interval (A, B). This solution is unique since:

$$M'(z) = \bar{F}(z)(bu(z) - 1)$$

and:

$$M''(z)|_{M'(z)=0} = \bar{F}(z)bu'(z) \geq 0$$

which means that $M(z)$ is U-shaped and has only one minimum on the interval $[A, B]$. Moreover, $M(A) = -A < 0$ for $A > 0$, and $M(B) = b - 1 > 0$, and if $A = 0$ then $M(0) = 0$ but $M'(0) < 0$. Hence $M(z)$ crosses the z-axes only once in the interval $(A, B]$ at $z = z_0$. Therefore, $M(z)$ is first negative up to $z = z_0$, then $M(z_0)$ attains 0 and then it is positive.

2. Additionally, the function in the numerator $L(z) \geq 0$ i.e. it is a non-decreasing function in the interval $[A, B]$ since $L'(z) \geq 0$ and $L(A) = h_cA \geq 0$. This implies the second statement of the lemma.

3. The third statement of the lemma follows from:

$$p_d(z) - w_d(z) - h \geq (p_d(z) - v - h)F(z) \geq 0$$

by using (5).

The proof is complete.

□

The lemma says that both the retail price and the wholesale price have a vertical asymptote and tend to infinity as z tends to z_0 from the right side. But at the same time the manufacturer's expected profit tends to 0 at this point because the order quantity decreases to 0, namely there is no demand, which is unrealistic. Therefore, the optimization is limited to the interval $[z_0, B]$, assuming that $\Pi_m(z_0) := 0$.

Finally, we obtain the following statement.

Theorem 4.3. *The problem:*

$$\max_{z \in S(z)} \Pi_m(z)$$

where $S(z) = [z_0, B] \cap \{z : w_d(z) \geq c\}$, *always has a solution which is not always unique.*

Proof. Since the function $\Pi_m(z)$ is continuous on the compact set $S(z)$, the statement follows from the Weierstrass Extreme Value Theorem (Rudin, 1953).

□

5 Numerical analysis

Due to the complexity of the manufacturers optimization problem, solutions are obtained numerically. The primary purpose of this numerical example is to illustrate the outcomes of the proposed Stackelberg game. Meaningful game parameters are reasonably chosen. Due to the mathematical complexity of the analytical results, we focus on sensitivity analysis with respect to the exchange handling costs h and h_c. The probability of product exchange is implicitly embedded in these parameter values. Let us assume that $a = 1$, $b = 2.2$, $c = 0.5$, $v = 0.2$, are specified, and that ε has a uniform distribution on $[A, B]$ with $A = 0$ and $B = 2$. The results are reported in Figures 1 and 2, Tables 2, 3, 4, and 5.

Based on numerical examples presented in the tables, we can state that the conclusions are similar in the case when the retailer's or consumer's exchange-related costs are variable. In the centralized case, optimal profits decline as exchange costs increase, even if there is an increase in inventory factor and retail price. In the decentralized channel, the increase of exchange costs affects the optimal profits of the manufacturer and the retailer in the same direction. It should be noticed here that in our case the optimal profit of the retailer is greater that the manufacturer's profit, which is not intuitive in the Stackelberg but can happen due to stochastic demand and the exchange costs used in the models.

Numerical simulations and Figures 1 and 2 further show that the profit of the channel both in the decentralized and the centralized case decreases as

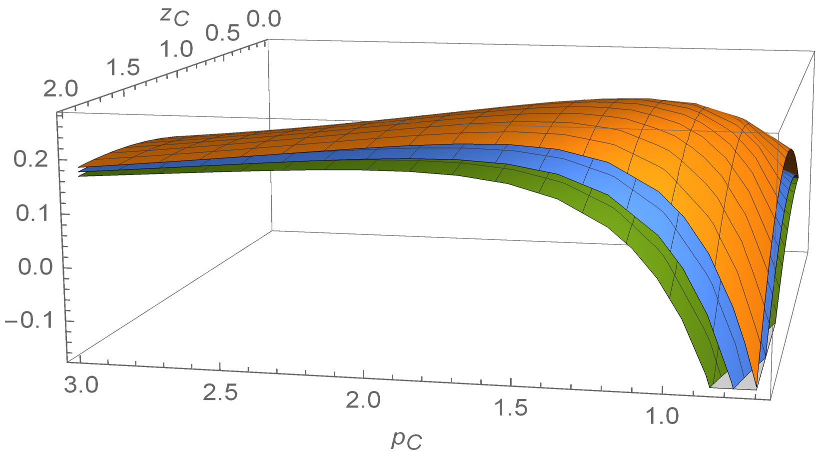

Figure 1: The expected profit with $h = 0, 0.1, 0.2$ (yellow, blue, green): centralized channel

Table 2: Sensitivity analysis with respect to h with $h_c = 0.1$: centralized channel

h	Π_c^*	p_c^*	z_c^*
0.1	0.2400	1.5156	1.5064
0.11	0.2368	1.53532	1.5103
0.12	0.2337	1.5550	1.5142
0.13	0.2306	1.5747	1.5180
0.14	0.2276	1.5944	1.5217
0.15	0.2247	1.614	1.5253
0.16	0.2218	1.6336	1.5289
0.17	0.2191	1.6532	1.5324
0.18	0.2163	1.6728	1.5359
0.19	0.2137	1.6924	1.5393
0.2	0.2111	1.7119	1.5427

exchange handling costs rise. Moreover, the decentralized profit is still less than the centralized one. This is in line with the fact that wholesale-price contracts generally fail to coordinate the supply chain.

Summarizing, we can state that allocating exchange handling costs to the retailer, who shares them with the consumer, significantly influences pricing decisions, inventory policies, and the overall performance of the supply chain. The profits of both the manufacturer and the retailer tend to decrease because the retailer faces both the higher wholesale price and the additional burden of exchange handling costs. The retailer increases the re-

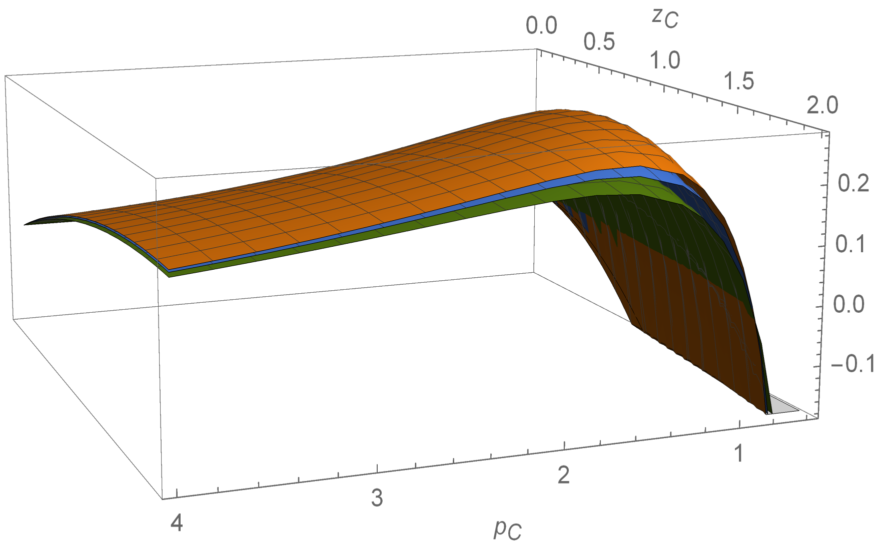

Figure 2: The expected profit with $h_c = 0, 0.05, 0.15$ (yellow, blue, green): centralized channel

Table 3: Sensitivity analysis with respect to h with $h_c = 0.1$: decentralized channel

h	Π_m^*	Π_r^*	Π_d^*	w_d^*	p_d^*	z_d^*
0.1	0.0629	0.1160	0.1789	0.9628	2.7705	1.3824
0.11	0.0620	0.1145	0.1765	0.9679	2.8046	1.3843
0.12	0.0612	0.1129	0.1741	0.9730	2.8387	1.3862
0.13	0.0604	0.1114	0.718	0.9780	2.8728	1.3880
0.14	0.0596	0.1100	0.1696	0.9831	2.9068	1.3898
0.15	0.0588	0.1086	0.1674	0.9881	2.9408	1.3916
0.16	0.0581	0.1072	0.1653	0.9932	2.9748	1.3933
0.17	0.0573	0.1059	0.1632	0.9982	3.0088	1.395
0.18	0.0566	0.1046	0.1612	1.0033	3.0427	1.3967
0.19	0.0559	0.1033	0.1592	1.0083	3.0766	1.3983
0.2	0.0552	0.1020	0.1572	1.0133	3.1105	1.3999

tail price and raises the stock factor to reduce the risk of shortages and cover the additional costs of exchange. Unsold products can then be returned for recovery, partially offsetting the losses but not fully compensating for the higher costs and reduced consumer demand. The combined effect of higher

Table 4: Sensitivity analysis with respect to h_c with $h = 0.1$: centralized channel

h_c	Π_c^*	p_c^*	z_c^*
0.01	0.2733	1.42709	1.4677
0.02	0.2692	1.4370	1.4723
0.03	0.2652	1.4469	1.4769
0.04	0.2613	1.4568	1.4813
0.05	0.2575	1.4667	1.4857
0.06	0.2538	1.4765	1.4900
0.07	0.2502	1.4863	1.4942
0.08	0.2468	1.4961	1.4984
0.09	0.2434	1.5059	1.5024
0.1	0.2400	1.5156	1.5064

Table 5: Sensitivity analysis with respect to h_c with $h = 0.1$: decentralized channel

h_c	Π_m^*	Π_r^*	Π_d^*	w_d^*	p_d^*	z_d^*
0.01	0.0717	0.1321	0.2038	0.9168	2.552	1.3634
0.02	0.0706	0.1301	0.2007	0.9219	2.5764	1.3657
0.03	0.0696	0.1282	0.1978	0.9270	2.6008	1.3680
0.04	0.0685	0.1263	0.1948	0.9322	2.6251	1.37017
0.05	0.0675	0.1245	0.1920	0.9373	2.6494	1.3723
0.06	0.0665	0.1227	0.1892	0.9424	2.6737	1.3744
0.07	0.0656	0.1209	0.1865	0.9476	2.6979	1.3765
0.08	0.0647	0.1193	0.1840	0.9527	2.7222	1.3785
0.09	0.0638	0.1176	0.1814	0.9577	2.7463	1.3805
0.1	0.0629	0.1160	0.1789	0.9628	2.7705	1.3824

wholesale and retail prices, along with a higher stock factor, leads to lower sales and greater inventory management challenges, further reducing the expected profits of both parties.

6 Conclusions

Product exchanges create significant costs for both manufacturers and retailers. These costs arise from handling returned items, logistics, and the risk of having unsold inventory. At the same time, exchanges provide value to customers by reducing the risk of purchase and strengthening their trust in the retailer. When exchanges are managed well, they can improve customer loyalty and contribute to higher long-term profits. When they are poorly managed, however, the associated costs quickly reduce margins (Han et al., 2017; Kalantary et al., 2023; www 1).

Our study is closely related to Liu et al. (2020) and Bieniek (2025), both of which analyze wholesale-price contracts with exchange channels. Unlike Liu et al. (2020), who treat retail price as exogenous, our model endogenously determines the retail price as part of the contract scenario. While Bieniek (2025) examines the wholesale-price contract with manufacturer-handled exchanges, we extend the analysis to retailer-handled ones.

Specifically, this paper investigates the retailer's role in managing returns under a contract modeled as a Stackelberg game with stochastic demand. We derive optimal prices and order quantities, showing that they are influenced by the exchange handling costs. The mathematical difficulty arising from the discontinuity of the functions of the retail and wholesale price was overcome by restricting the set of results to those that are realistic. This difficulty stemmed from the choice of a rapidly changing power and, moreover, the multiplicative demand with the effect of scale and also from the introduction of exchange handling costs. A numerical example confirms that return-related costs substantially affect the optimal solutions, sometimes leading to retailers' profits being greater than the manufacturers' profits, which is not obvious in a Stackelberg game.

Future research can extend this framework to alternative coordinating contracts, multi-period settings, or models where the exchange fraction is a decision variable. Incorporating environmentally sustainable practices into exchange handling also represents a promising direction. The practical implications of our work offer guidance to managers designing wholesale-price contracts in environments where product exchanges are significant, i.e. in electronic commerce.

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